

Invention of Numbers

- One plus one is two.
- There are 60 seconds in one minute.
- 60 minutes equals 1 hour.
- There are 24 hours in 1 day.
- One year consists of 365 days.



It all began with COUNTING

- Evidence points to the idea that numbers and counting began with the number one.
- Evidence that this occurred as long as 20,000 years ago.
- An ancient artifact known as the Ishango Bone found in Africa in 1950.



- Your first experience with numbers was when you learned to count.

Natural Numbers → 1, 2, 3, 4, . . .

- Historians believe numbers and counting expanded beyond one around 4,000 B.C. in Sumeria.
- One of the first civilizations to feature cities that were centres of trade, the people of Sumeria needed new methods of counting and record keeping.

Classifying Numbers

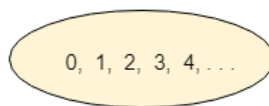
Natural Numbers → 1, 2, 3, 4, . . .

The first recorded use of a zero-like symbol dates to sometime around the third century B.C.

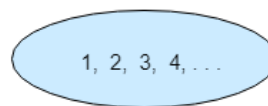
Whole Numbers → 0, 1, 2, 3, 4, . . .

The Natural and Whole numbers are a collection that can be represented using a diagram.

- An ellipsis (. . .) shows that the pattern continues and all numbers in the pattern are part of the collection.
- We can use an oval to indicate these numbers belong to a collection.



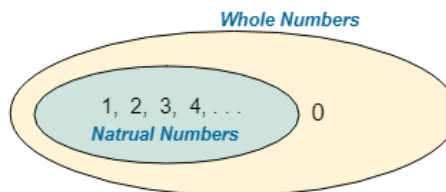
Whole Numbers



Natural Numbers

Are all Natural numbers Whole numbers? **YES!**

Are all Whole numbers Natural numbers? **NO!**



All numbers inside the Whole numbers oval are "Whole Numbers"

This diagram illustrates that all Natural numbers are also Whole numbers.

Negative Numbers

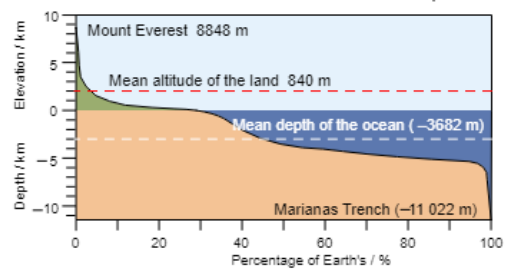
Before 0 started to be used as a number, many cultures understood the concept of negative numbers.

About 200 B.C. negative numbers appeared in the Ancient Chinese writing the Nine Chapters on the Mathematical Art.

Positive Numbers									
0	1	2	3	4	5	6	7	8	9
	-	=	≡	≡	≡	⊥	⊥	⊥	⊥
Negative Numbers									
0	-1	-2	-3	-4	-5	-6	-7	-8	-9
	-	=	≡	≡	≡	⊥	⊥	⊥	⊥

Why do we need negative numbers?

Altitude: Tells us how far above or below sea level a place is.



Spend \$50 on supplies

What is the profit?

The integer -5 best represents a negative profit (loss) of \$5.



Earn \$45 in sales

You need a negative number to describe your profit.

Integers

... -4, -3, -2, -1, 0, 1, 2, 3, 4 ...

Classifying Numbers

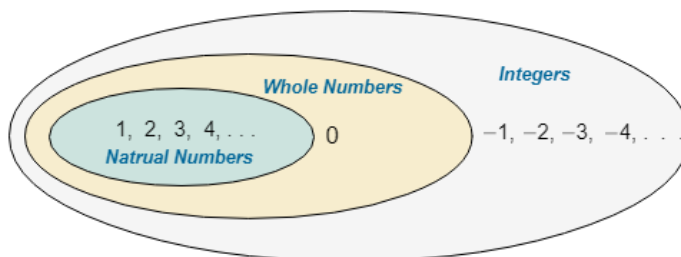
Natural Numbers → 1, 2, 3, 4, ...

Whole Numbers → 0, 1, 2, 3, 4, ...

Integers → ... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

Are all Whole numbers Integers? **YES!**

Are all Integers, Whole numbers? **NO!**



Place a check mark where you think which category(s) 11 belongs to.

- Natural Number
- Whole Number
- Integer

Place a check mark where you think which category(s) -15 belongs to.

- Natural Number
- Whole Number
- Integer

Types of Integers

Positive Integers → 1, 2, 3, 4, ...

Positive integers can be represented with or without the positive sign in front of the number.

For example: +9 and 9 both represent positive 9.

Zero → 0

Zero has no sign because it is neither positive nor negative.

Negative Integers → -1, -2, -3, -4, ...

Negative integers **must** be represented with a negative sign in front of the number.

For example: -15 is the only way to write negative 15.

The Number System

Check your Understanding

Question 1

Identify the number category that each of the following numbers belong to by marking an “X” in the appropriate box.

	Number	Natural Numbers	Whole Numbers	Integers
1	43			
2	-32			
3	0			
4	22			
5	14,000			
6	-14,000			
7	7			
8	-7			
9	-2			
10	-14,000,091			
11	34			
12	-1			
13	66			
14	54			

15	890			
16	-568			
17	17			
18	22			
19	-33			
20	33			

Answer Key

Answer 1

	Number	Natural Numbers	Whole Numbers	Integers
1	43	X	X	X
2	-32			X
3	0		X	X
4	22	X	X	X
5	14,000	X	X	X
6	-14,000			X
7	7	X	X	X
8	-7			X
9	-2			X
10	-14,000,091			X
11	34	X	X	X
12	-1			X
13	66	X	X	X
14	54	X	X	X
15	890	X	X	X

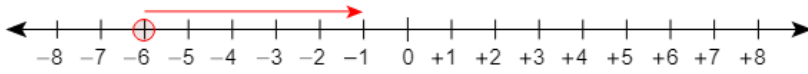
16	-568			X
17	17	X	X	X
18	22	X	X	X
19	-33			X
20	33	X	X	X

Adding Integers on a Number Line

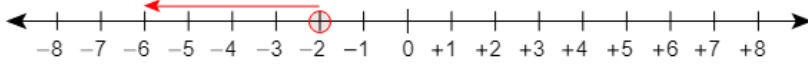
To ADD a positive number we move to the right. \longrightarrow

To ADD a negative number we move to the left. \longleftarrow

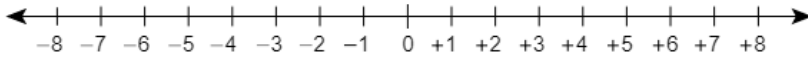
Add the following: $(-6) + (+5) = -1$



Add the following: $(-2) + (-4) = -6$



Add the following: $(-5) + (+7) = +2$ **Show Me**



Integers as Tiles

Representing Integers using Tiles

Positive numbers we will use a white tile.

Negative numbers we will use a black tile.

= +1

= -1

Can use tiles to represent integers.

+2 =

-4 =

What happens when you place a positive tile with it's opposite?

= 0

This is called the: **Zero Principle**

= 0

Each pair of opposites creates a 0.

= 0

Zero Principle : states that when you have **equal** number of **opposite** integers, they sum to be **zero** .

Using the zero principle, show the value of -3 in more than one way.

= -3


Your turn.


Give another way using the zero principle to represent -3

Another way.

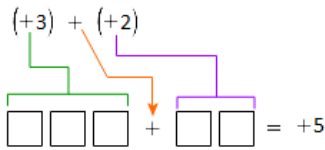
Another way.

Adding Integers using Tiles

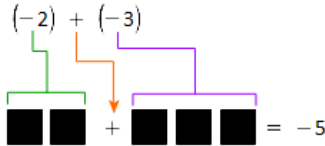
Positive numbers we will use a white tile. 

Negative numbers we will use a black tile. 

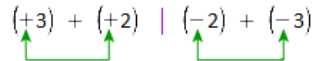
Add the following using tiles.



Add the following using tiles.



Notice we are adding integers with the same sign.

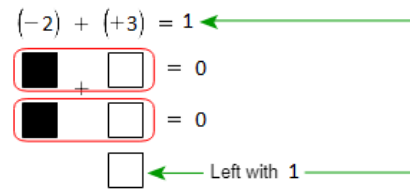


In your own words describe how to add integers which have the same sign.

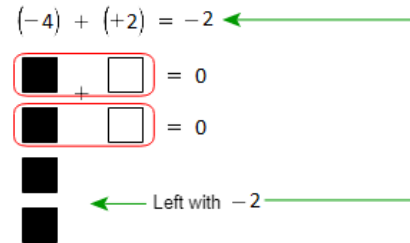
When you add tiles that are the same sign, you add the number of tiles.

Look what happens when the signs are different.


Add the following using tiles.




Add the following using tiles.



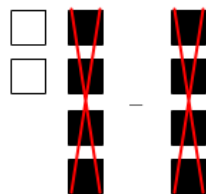
Subtracting Integers using Tiles

Positive numbers we will use a white tile. 

Negative numbers we will use a black tile. 

Subtract the following using tiles.

$(-2) - (-4) = +2$



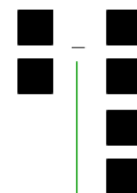
Now we can take away (subtract) 4 negative tiles.

Leaves us with 2 white tiles.

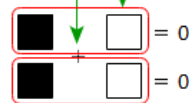
The above method is the LONG way.

A shorter method.

$(-2) - (-4) = +2$



Add the opposite.





Leaves us with 2 white tiles.

$(-2) - (-4) = +2$

$(-2) + (+4) = +2$

Subtracting Integers using Tiles

Positive numbers we will use a white tile. 

Negative numbers we will use a black tile. 

Use "add the opposite" and tiles to answer the following questions.

a) $(-4) - (-5)$

Answer **Solution**

b) $(-4) - (+3)$

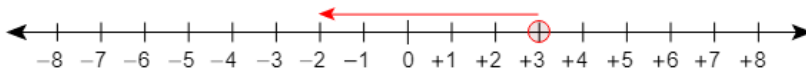
Answer **Solution**

Subtracting Integers using the Number Line

Subtract the following using the number line.

$$(+3) - (+5) = -2$$

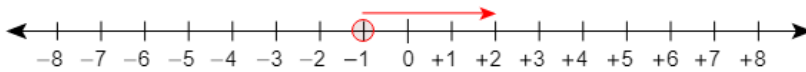
$$\begin{array}{c} \downarrow \quad \downarrow \\ (+3) + (-5) \end{array}$$



Subtract the following using the number line.

$$(-1) - (-3) = +2$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ (-1) + (+3) \end{array}$$

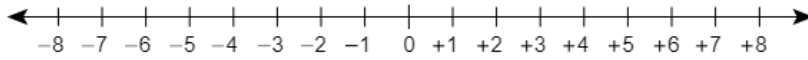


Subtracting Integers using the Number Line

Subtract the following using the number line.

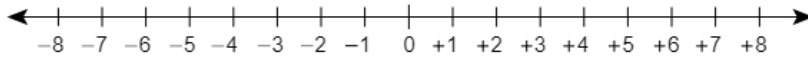
a) $(-5) - (+3)$

Answer **Solution**



b) $(+2) - (-6)$

Answer **Solution**



Adding and Subtracting Integers

Types of Integers

Positive Integers \rightarrow 1, 2, 3, 4, . . .

Positive integers can be represented with or without the positive sign in front of the number.

For example: +9 and 9 both represent positive 9.

$$(+5) + (+3) \rightarrow 5 + 3$$

$$(+2) - (+6) \rightarrow 2 - 6$$

Zero \rightarrow 0

Zero has no sign because it is neither positive nor negative.

Negative Integers \rightarrow -1 -2 -3 -4, . . .

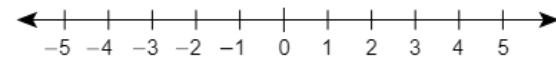
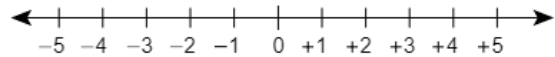
Negative integers **must** be represented with a negative sign in front of the number.

For example: -15 is the only way to write negative 15.

$$(+9) - (-2) \rightarrow 9 - (-2)$$

Do not write it like this: $9 - - 2$

$$-5 - (+6) \rightarrow -5 - 6$$



Add or subtract the following integers. Can use the number line if needed.

a) $-2 + (-3)$

Answer **Solution**

b) $-2 - 3$

Answer **Solution**

c) $-4 + 5$

Answer **Solution**

d) $-3 - (-2)$

Answer **Solution**

e) $(-4) - (+1)$

Answer **Solution**

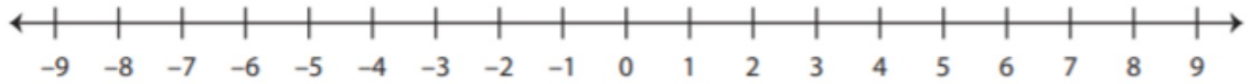
Adding and Subtracting Integers

Check your Understanding

Question 1

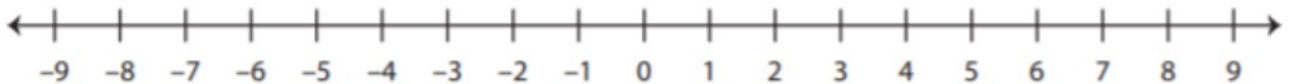
|

1) $4 + (-5) =$ _____



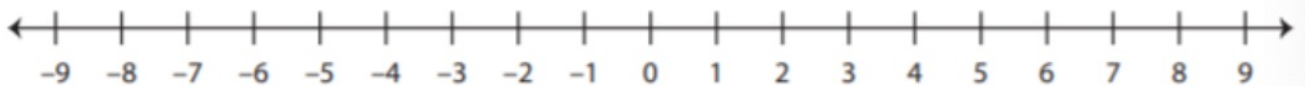
Question 2

2) $-7 + 2 =$ _____



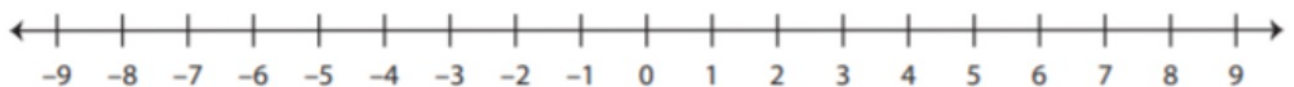
Question 3

3) $-3 + (-4) =$ _____



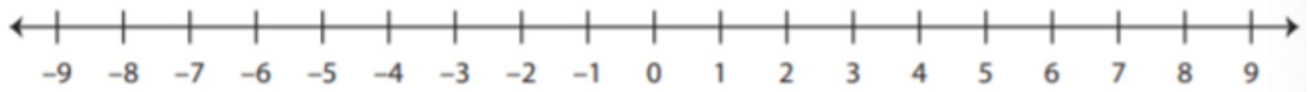
Question 4

4) $1 + 7 =$ _____



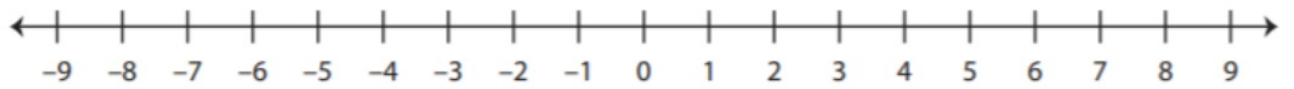
Question 5

5) $3 + (-12) =$ _____



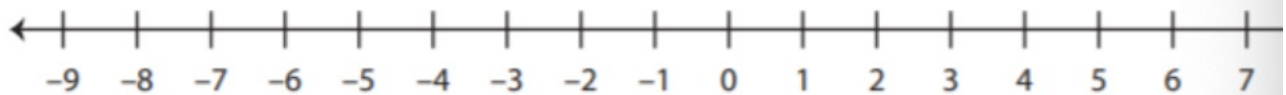
Question 6

6) $1 - 10 =$ _____



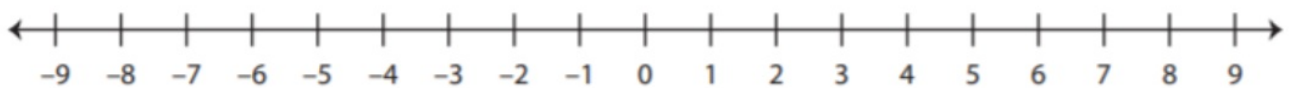
Question 7

7)
 $0 - (-7) =$ _____



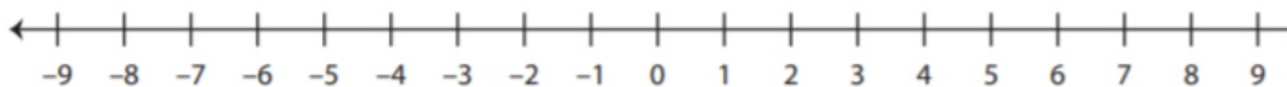
Question 8

8)
 $-6 - 2 =$ _____



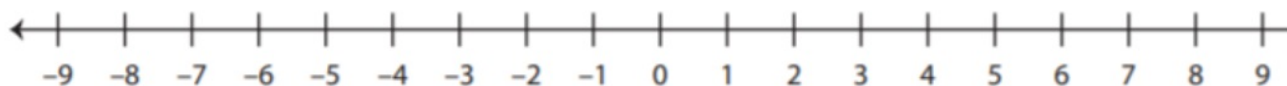
Question 9

9) $5 - 4 = \underline{\hspace{2cm}}$



Question 10

10) $-8 - (-3) = \underline{\hspace{2cm}}$



Question 11

Write the addition statement represented with each tile model. Then find the sum. Each white tile equals +1, and each black tile represents -1.



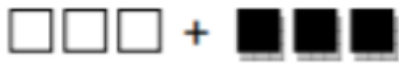
Question 12

Write the addition statement represented with each tile model. Then find the sum. Each white tile equals +1, and each black tile represents -1.

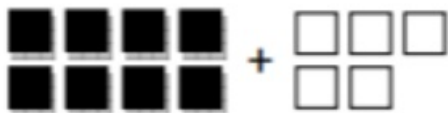


Question 13

Write the addition statement represented with each tile model. Then find the sum. Each white tile equals +1, and each black tile represents -1.

**Question 14**

Write the addition statement represented with each tile model. Then find the sum. Each white tile equals +1, and each black tile represents -1.

**Question 15**

$$5 + (-4) =$$

Question 16

$$4 + (-12) =$$

Question 17

$$(-5) + 5 =$$

Question 18

$$10 + (-5) =$$

Question 19

$$-3 + 5 =$$

Question 20

$$-2 + 5 + (-1) =$$

Question 21

$$2 - 5 =$$

Question 22

$$-5 - 5 =$$

Question 23

$$7 - (-2) =$$

Question 24

$$12 - (-12) =$$

Question 25

$$-4 - (-5) =$$

Question 26

$$0 - (-3) =$$

Answer Key

Answer 1

-1

Answer 11

$$3 + -4 = -1$$

Answer 22

-10

Answer 2

-5

Answer 12

$$-6 + 4 = -2$$

Answer 23

9

Answer 3

-7

Answer 13

$$3 + (-3) = 0$$

Answer 24

24

Answer 4

8

Answer 14

$$-8 + 5 = -3$$

Answer 25

1

Answer 5

-9

Answer 15

1

Answer 26

3

Answer 6

-9

Answer 16

-8

Answer 7

7

Answer 17

0

Answer 8

-8

Answer 18

5

Answer 9

1

Answer 19

2

Answer 10

-5

Answer 20

2

Answer 21

-3

Multiplying Integers

What is this equal to?

$$(+5) + (+5) + (+5) + (+5) + (+5) + (+5) = +30$$

Is there a quicker way to get the answer?

$$6 \times (+5) = +30$$



$$(+6) \times (+5) = +30$$

$$(\text{positive number}) \times (\text{positive number}) = \text{positive number}$$

$$6 \times 5 = 30$$

When you see no sign in front of the number then you know it is a positive number.

Rules for Multiplying Integers

$$(\text{positive number}) \times (\text{positive number}) = \text{positive number}$$

$$(\text{positive number}) \times (\text{negative number}) = \text{negative number}$$

$$(\text{negative number}) \times (\text{positive number}) = \text{negative number}$$

What is this equal to?

$$(-5) + (-5) + (-5) + (-5) + (-5) + (-5)$$

$$-10 + (-5) + (-5) + (-5) + (-5)$$

$$-15 + (-5) + (-5) + (-5)$$

$$-20 + (-5) + (-5)$$

$$-25 + (-5) = -30$$

Is there a quicker way to get the answer?

$$6 \times (-5) = -30$$



$$(+6) \times (-5) = -30$$

$$(\text{positive number}) \times (\text{negative number}) = \text{negative number}$$

$$2 \times 3 = 3 \times 2$$

$$6 = 6$$

$$(+6) \times (-5) = (-5) \times (+6)$$

$$(\text{negative number}) \times (\text{positive number}) = \text{negative number}$$

Applications of Multiplying Integers

Using integers create a multiplication statement to represent the change for each of the following situations and then give the answer.

- 1) The temperature falls 3°C each hour for 8 hours. What is the total change in temperature over the eight-hour period?

$$-3^{\circ}\text{C} \times 8 = -24^{\circ}\text{C}$$

$$8 \times (-3^{\circ}\text{C}) = -24^{\circ}\text{C}$$

$$(8)(-3^{\circ}\text{C}) = -24^{\circ}\text{C}$$

multiplication

The total change in temperature over the eight-hour period is -24°C .

- 2) Gurminder earns $\$12.00$ interest in an investment each month for 6 months. How much interest will Gurminder have earned at the end of six months?

$$(+12) \times (+6) = +72$$

$$12 \times 6 = 72$$

$$(+12)(+6) = +72$$

$$(12)(6) = 72$$

Gurminder earns $\$72.00$ in interest over the 6 month period.

Applications of Multiplying Integers

Using integers, create a multiplication statement to represent the change for each of the following situations and then give the answer.

- 1) A stock loses \$1.50 each day over a five-day period. What is the total change in the value of the stock at the end of five days?

Answer **Solution**

- 2) You are flying a single engine light airplane. It is climbing at 750 feet per minute for 20 minutes. What is your altitude at the end of 20 minutes?

Answer **Solution**

Multiplying Integers

$$(+4)(-5) = -20$$

$$(4)(-5) = (-5) + (-5) + (-5) + (-5) \\ = -20$$

Rules for Multiplying Integers

$$(+)\times(+)=+$$

$$(+)\times(-)=-$$

$$(-)\times(+)= -$$

$$(-)\times(-)=+$$

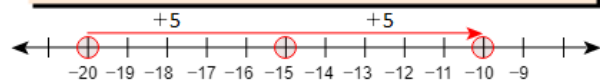
$$(-4)(-5) = +20$$

	$(+4) \times (-5) = -20$	$+5$
	$(+3) \times (-5) = -15$	$+5$
	$(+2) \times (-5) = -10$	$+5$
	$(+1) \times (-5) = -5$	$+5$
Decrease	$0 \times (-5) = 0$	$+5$
zero	$(-1) \times (-5) = +5$	$+5$
by 1	$(-2) \times (-5) = +10$	
	$(-3) \times (-5) = +15$	
	$(-4) \times (-5) = +20$	

This pattern suggest you can get the answer to the multiplication by adding 5 to the previous answer.

What can you conclude when we multiply a negative number by a negative number?

$$(\text{negative number}) \times (\text{negative number}) = \text{positive number}$$



Multiplying Integers

Using integers, create a multiplication statement to represent the change for each of the following situations and then give the answer.

- 1) Michaela travels to a different country and her cell phone company charges her \$2 for every text message. She discovers on her next billing statement they charged her for 5 extra text messages. Michaela contacts them and wants the charges to be removed. What was the charge to her finances due to them removing the charges?

$$(+5) \times (-2) = -10\$$$

↑ The cell phone company has to take away the 5 groups of -2

$$(-5) \times (-2) = +10\$ \leftarrow \text{means 10 dollars has been added to your account.}$$

- 2) Today a fresh water tank has 10,000 litres, and 500 litres has been taken out every day. What was the amount of water in the tank 3 days ago?

$$-3 \times (-500) = +1500$$

$$10,000 + 1500 = +11,500$$

Three days ago there was 11,500 litres of fresh water in the tank.

Multiplying Integers

$$(-3)(-4) = +12 \leftarrow \text{multiplying 2 negatives answer is POSITIVE}$$

$$(-3)(-4)(-1) = -12 \leftarrow \text{multiplying 3 negatives answer is NEGATIVE}$$

$$(-3)(-4)(-1)(-1) = +12 \leftarrow \text{multiplying 4 negatives answer is POSITIVE}$$

$$(-3)(-4)(-1)(-1)(-1) = -12 \leftarrow \text{multiplying 5 negatives answer is NEGATIVE}$$

If we multiplied 6 negatives would the answer be POSITIVE or NEGATIVE? \rightarrow answer would be POSITIVE

Even number of negatives (2, 4, 6, 8, ...) then the answer will be a POSITIVE number.

If we multiplied 7 negatives would the answer be POSITIVE or NEGATIVE? \rightarrow answer would be NEGATIVE

Odd number of negatives (3, 5, 7, 9, ...) then the answer will be a NEGATIVE number.

Example

$$\begin{array}{ccccccccc} (-2)(-4)(-2)(-5)(-3) & = & -240 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & & & \\ 1 & 2 & 3 & 4 & 5 & & & & \end{array}$$

There are 5 negatives.

The answer must be negative.

Dividing Integers

Division is the inverse of multiplication.

$$6 \times 4 = 24 \quad \text{then} \quad 24 \div 4 = 6$$

$$\text{and} \quad 24 \div 6 = 4$$

The same idea is true for integers.

$$(+5)(-4) = -20 \quad \text{then} \quad (-20) \div (+5) = -4$$

$$\text{and} \quad (-20) \div (-4) = +5$$

Negative number divided by a positive number = negative number

Positive number divided by a negative number = negative number

Negative number divided by a negative number = positive number

Positive number divided by a positive number = positive number

Example Simplify the following.

a) $(-18) \div (+3) =$
Answer Solution

b) $\frac{-42}{-7} =$
Answer Solution

c) $\frac{(-4)(10)}{-8} =$
Answer Solution

Dividing Integers

Multiply before dividing.

Simplify: $\frac{(-2)(-15)}{(-3)(5)}$ ← means division

multiplication

multiplication

RULE: Perform the math operation in the top, then the bottom and lastly divide.

Negative × Negative

$$\frac{(-2)(-15)}{(-3)(5)} = \frac{+30}{-15} = -2$$

Negative × Positive

Example

Simplify and do not answer as a decimal.

$$\frac{(-2)(5)}{(-3)(1)} = \frac{-10}{-3} = -10 \div (-3) = 3.3333 \dots$$

We don't want this answer.
Mathematically the answer is correct, BUT

$$= \frac{-10}{-3} \quad \text{You are not done!} \quad \text{Negative} \div \text{Negative}$$

$$= +\frac{10}{3} \quad \text{Do not have to write the positive symbol in our answer.}$$

$$= \frac{10}{3} \quad \text{Answer written with no sign in front, then it is understood to be positive!}$$

Example

Simplify and do not answer as a decimal.

$$\frac{(-3)(4)}{0 + (-7)}$$
Answer Solution

Multiplying and Dividing Integers

Check your Understanding

Question 1

$$(-5) \times 4 =$$

Question 2

$$(-12) \times (-3) =$$

Question 3

$$7 \times (-8) =$$

Question 4

$$(-6) \times (+3) \times 2 =$$

Question 5

$$8 \times (-9) \times 2 =$$

Question 6

$$(-2) \times (-4) \times (-5) =$$

Question 7

Explain why the product of two negative integers is always greater than the sum of the same two integers.

Question 8

Find a pair of integers whose

- Sum is -5 and product is -24 .
- Sum is -7 and product is 12 .
- Sum is 4 and product is -5 .
- Difference is 7 and product is -10 .

Question 9

From sea level, a submarine descends 20 m per minute. Where is the submarine in relation to sea level after 1 minutes?

Question 10

The temperature falls 3°F each hour for 8 hours. What is the total change in temperature over this time?

Question 11

Abdul earns \$12.00 in interest each month for eight months. How much interest did he earn over this time?

Question 12

The temperature drops 2°C each hour from 6:00 p.m. to 3:00 a.m. What is the total change in temperature during this period?

Question 13

Felix reported that the coldest day on record for his town was five times colder than yesterday's temperature, 4°C . What was the temperature of the coldest day on record in Felix's town?

Question 14

A local theater reported losses of \$375 each day for three days. What was the total loss for the three days?

Question 15

A football team lost 8 yards on every play for 5 plays. What integer would represent the total number of yards the team lost on these 5 plays?

Question 16

Desmond's monthly Netflix subscription costs \$10. If he charges it to his credit card each month for 5 months without making any payments, what will his debt be?

Question 17

At 7:00 pm the temperature was 22°F . If the temperature drops 3°F per hour, what was the temperature at 11:00 pm?

Question 18

$$(-28) \div (-7) =$$

Question 19

$$20 \div (-10) =$$

Question 20

$$(-91) \div 13 =$$

Question 21

$$\frac{(-24)}{(-8)} =$$

Question 22

$$\frac{(-10)(12)}{(3)} =$$

Question 23

$$\frac{(-3)(-15)}{(-9)} =$$

Question 24

$$\frac{(-8)(12)}{(-4)(-6)} =$$

Question 25

$$\frac{(-22)(-8)}{(-11)(-4)} =$$

Question 26

$$\frac{(-9)(12)}{(-3)} =$$

Question 27

$$\frac{(-2)(-8)(-5)(-9)}{(-6)(-4)(-10)} =$$

Question 28

$$\frac{(-2)(-16)(-30)}{(-5)(-4)(-12)} =$$

Question 29

$$\frac{(-20)(-8)(-5)(0)}{(-2)(-4)(-3)} =$$

Answer Key

Answer 1

-20

Answer 2

36

Answer 3

-56

Answer 4

-36

Answer 5

-144

Answer 6

-40

Answer 7

Product of two negative integers is always positive. Adding two negative integers together will always result in a negative integer. Positive integer is greater than a negative integer.

Answer 8

- a. -8 and 3
- b. -3 and -4
- c. -1 and 5
- d. -5 and 2

Answer 9

$$-20 \times 15 = -300\text{m}$$

The submarine is 300 m below sea level.

Answer 10

$$-3 \times 8 = -24$$

Answer 11

$$\$12 \times 8 = \$96$$

Answer 12

$$-2 \times 9 = -18^{\circ}\text{C}$$

The total change is 18°C lower.

Answer 13

$$-4 \times 5 = -20$$

Answer 14

$$-\$375 \times 3 = -\$1,125$$

Answer 15

$$-8 \times 5 = -40 \text{ yards}$$

Answer 16

$$-\$10 \times 5 = -\$50$$

Answer 17

$$-3 \times 4 = -12$$

$$22 + (-12) = 10^{\circ}\text{C}$$

Answer 18

4

Answer 19

-2

Answer 20

-7

Answer 21

3

Answer 22

-40

Answer 23

-5

Answer 24

-4

Answer 25

4

Answer 26

36

Answer 27

-3

Answer 28

4

Answer 29

0

Order of Operations

Three students won a contest and are given a skill testing question. If answered correctly they will win a prize.

$$4 + 3 \times 2 - 1$$

Sammy



$$\begin{aligned} 4 + 3 \times 2 - 1 \\ 7 \times 2 - 1 \\ 14 - 1 \\ 13 \end{aligned}$$

Alexis



$$\begin{aligned} 4 + 3 \times 2 - 1 \\ 4 + 6 - 1 \\ 10 - 1 \\ 9 \end{aligned}$$

Cameron



$$\begin{aligned} 4 + 3 \times 2 - 1 \\ 7 \times 2 - 1 \\ 7 \times 1 \\ 7 \end{aligned}$$

Who is correct?

Have to determine an order in which these math calculations have to be done.

BEDMAS

Order of Operations

Brackets

$$3 \cdot 2 + 4 \quad 3 \cdot (2 + 4)$$



The "dot" means multiplication.

$$3 \cdot 2 + 4 \quad 3 \cdot (2 + 4)$$

- ▶ Both expressions have the same numbers: 3, 2, and 4.
- ▶ Both expressions have the same math operations.
 - multiplication
 - addition
- ▶ **Order of Operations** tells us to complete the calculations inside the brackets first.
- ▶ The inclusion of brackets into the expression changes the outcome of the expression.

Exponents

Exponents are used for repeated multiplication.

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5 \quad \leftarrow \text{how many times 3 is multiplied.}$$

$$3^5 \quad \leftarrow \text{exponent}$$

↑
base

This is read as "3 to the power 5"
or
"3 to the fifth"
or
"3 to the exponent 5"

Example

Write the following in exponential form. (write using exponents)

$$2 \cdot 2 \cdot 2 \cdot (-3)(-3)(-3)(-3)$$

$$2^3 \cdot (-3)^4$$

Example

Simplify:

- a) $(-2)^4$ ← the exponent belongs to the brackets everything inside the brackets is multiplied by itself including the negative sign.

$$\begin{aligned} (-2)^4 &= (-2)(-2)(-2)(-2) = +16 \\ &\text{even number of negatives} \end{aligned}$$

- b) -2^4 ← the exponent belongs to the base and **NOT** the negative sign. Only the base is multiplied by itself. There is only one negative sign.

$$\begin{aligned} -2^4 &= -1 \times 2 \times 2 \times 2 \times 2 \\ &= -1 \times 16 \\ &= -16 \end{aligned}$$

BEDMAS

Order of Operations

- 1) Calculate all operations in brackets first.
- 2) Simplify all expressions with exponents.
- 3) Complete division and multiplication as they occur, working from left to right.
- 4) Complete addition and subtraction as they occur, working from left to right.

How can you remember the order of operations?

BEDMAS

Brackets

Exponents

Division

Multiplication

Addition

Subtraction

If division and multiplication appear together perform operation from left to right.

If they appear together perform the operation from left to right.

Example Simplify

$$\begin{aligned} 24 \div 6 \times 5 \div 2 \\ 4 \times 5 \div 2 \\ 20 \div 2 \\ 10 \end{aligned}$$

Example Simplify

$$\begin{aligned} (5 \cdot 2 - 5 + 4) \div 3 \\ (10 - 5 + 4) \div 3 \\ (5 + 4) \div 3 \\ 9 \div 3 \\ 3 \end{aligned}$$

Order of Operations

Example Simplify

$$-3[2(3-5)^3 + (4-2)^2]$$

$$-3[2(-2)^3 + (2)^2]$$

$$-3[2(-8) + 4]$$

$$-3[-16 + 4]$$

$$-3[-12]$$

36

Example Simplify the following using BEDMAS.

a) $[-2(8 \div 4 + 3) - 5(13 - 9)] \div 6$

Answer **Solution**

b) $-2^3(5 - 6) + (4 - 12 \div 2)$

Answer **Solution**

c) $(-3)^2(2^2 - 11) - (4^2 - 13)$

Answer **Solution**

d) $-4^2 \cdot 10^2 + 3^3 \cdot 10^3 - 6^2 \cdot 10$

Answer **Solution**

Order of Operations

Check your Understanding

Question 1

$$(-2) + 9 \times 10 =$$

Question 2

$$(-7) - 3^2 =$$

Question 3

$$3 \times (8 + (-2)) =$$

Question 4

$$5 + (-9) \times 9 =$$

Question 5

$$(-8) \times (-6) - (-5)^2 =$$

Question 6

$$3 \times [9 + (-8)]^2 =$$

Question 7

$$-2^2 \times (-9) - 9 =$$

Question 8

$$10 \times (-5) + (-6)^2 =$$

Question 9

$$9 \times [(-3) + 4 - (-2)^2] =$$

Question 10

$$(-2)^2 \div (-4) + 4 \times 9 =$$

Question 11

$$(-6) \times [(-5) + (-9) - (-2)^3] =$$

Question 12

$$(-7) - 5^2 + (-6) \times (-8) =$$

Question 13

$$2 \times [(-5) + 6 - (-7)] \div (-2)^2 =$$

Question 14

$$[(-10) \times 9] \div (-9) + 10 - 4^2 =$$

Question 15

$$(-4)^3 - (-8) \times [5 + 6 \div (-3)] =$$

Question 16

$$\{(-3)^2 \times [3 - (-7) + (-10)]^2\} \div 7 =$$

Question 17

$$\{(-7)^2 \div [3 - (-4)]^2\} \times [7 + (-6)] =$$

Question 18

$$[(-3)^3 - (-5)] \times \{(-8) \div [5 + (-7)]^2\} =$$

Answer Key

Answer 1

$$\begin{aligned} & (-2) + \underline{9 \times 10} \\ & = \underline{(-2) + 90} \\ & = 88 \end{aligned}$$

Answer 2

$$\begin{aligned} & (-7) - \underline{3^2} \\ & = \underline{(-7) - 9} \\ & = -16 \end{aligned}$$

Answer 3

$$\begin{aligned} & 3 \times (\underline{8 + (-2)}) \\ & = \underline{3 \times 6} \\ & = 18 \end{aligned}$$

Answer 4

$$\begin{aligned} & 5 + \underline{(-9) \times 9} \\ & = \underline{5 + (-81)} \\ & = -76 \end{aligned}$$

Answer 5

$$\begin{aligned} & (-8) \times (-6) - \underline{(-5)^2} \\ & = \underline{(-8) \times (-6)} - 25 \\ & = \underline{48 - 25} \\ & = 23 \end{aligned}$$

Answer 6

$$\begin{aligned} & 3 \times (\underline{9 + (-8)})^2 \\ & = 3 \times \underline{1^2} \\ & = \underline{3 \times 1} \\ & = 3 \end{aligned}$$

Answer 7

$$\begin{aligned} & -2^2 \times (-9) - 9 \\ & = \underline{-4 \times (-9)} - 9 \\ & = \underline{36 - 9} \\ & = 27 \end{aligned}$$

Answer 8

$$\begin{aligned} & 10 \times (-5) + \underline{(-6)^2} \\ & = \underline{10 \times (-5)} + 36 \\ & = \underline{(-50) + 36} \\ & = -14 \end{aligned}$$

Answer 9

$$\begin{aligned} & 9 \times ((-3) + 4 - \underline{(-2)^2}) \\ & = 9 \times (\underline{(-3) + 4 - 4}) \\ & = 9 \times \underline{(1 - 4)} \\ & = \underline{9 \times (-3)} \\ & = -27 \end{aligned}$$

Answer 10

$$\begin{aligned} & \underline{(-2)^2} \div (-4) + 4 \times 9 \\ & = \underline{4 \div (-4)} + 4 \times 9 \\ & = (-1) + \underline{4 \times 9} \\ & = \underline{(-1) + 36} \\ & = 35 \end{aligned}$$

Answer 11

$$\begin{aligned} & (-6) \times ((-5) + (-9) - \underline{(-2)^3}) \\ & = (-6) \times (\underline{(-5) + (-9) - (-8)}) \\ & = (-6) \times (\underline{(-14) - (-8)}) \\ & = \underline{(-6) \times (-6)} \\ & = 36 \end{aligned}$$

Answer 12

$$\begin{aligned} & (-7) - \underline{5^2} + (-6) \times (-8) \\ & = (-7) - 25 + \underline{(-6) \times (-8)} \\ & = \underline{(-7) - 25} + 48 \\ & = \underline{(-32) + 48} \\ & = 16 \end{aligned}$$

Answer 13

$$\begin{aligned} & 2 \times (\underline{(-5) + 6 - (-7)}) \div (-2)^2 \\ & = 2 \times (\underline{1 - (-7)}) \div (-2)^2 \\ & = 2 \times 8 \div \underline{(-2)^2} \\ & = \underline{2 \times 8} \div 4 \\ & = \underline{16 \div 4} \\ & = 4 \end{aligned}$$

Answer 14

$$\begin{aligned} & \left(\frac{(-10) \times 9}{1} \right) \div (-9) + 10 - 4^2 \\ & = (-90) \div (-9) + 10 - 4^2 \\ & = \frac{(-90)}{(-9)} + 10 - 16 \\ & = \frac{10 + 10}{1} - 16 \\ & = \frac{20 - 16}{1} \\ & = 4 \end{aligned}$$

Answer 15

$$\begin{aligned} & (-4)^3 - (-8) \times \left(5 + \frac{6}{(-3)} \right) \\ & = (-4)^3 - (-8) \times \left(\frac{5 + (-2)}{1} \right) \\ & = \frac{(-4)^3}{1} - (-8) \times 3 \\ & = \frac{(-64)}{1} - \frac{(-8) \times 3}{1} \\ & = \frac{(-64) - (-24)}{1} \\ & = -40 \end{aligned}$$

Answer 16

$$\begin{aligned} & \{(-3)^2 \times [3 - (-7) + (-10)]^2\} \div 7 \\ & = \{(-3)^2 \times [0]^2\} \div 7 \\ & = \{9 \times 0\} \div 7 \\ & = 0 \div 7 \\ & = 0 \end{aligned}$$

Answer 17

$$\begin{aligned} & \left((-7)^2 \div \left(\frac{3 - (-4)}{1} \right)^2 \right) \times (7 + (-6)) \\ & = \left(\frac{(-7)^2}{7^2} \right) \times (7 + (-6)) \\ & = (49 \div 7^2) \times (7 + (-6)) \\ & = \frac{49 \div 49}{1} \times (7 + (-6)) \\ & = 1 \times \left(\frac{7 + (-6)}{1} \right) \\ & = \frac{1 \times 1}{1} \\ & = 1 \end{aligned}$$

Answer 18

$$\begin{aligned} & \left(\frac{(-3)^3}{1} - (-5) \right) \times \left((-8) \div (5 + (-7))^2 \right) \\ & = \left(\frac{(-27) - (-5)}{1} \right) \times \left((-8) \div (5 + (-7))^2 \right) \\ & = (-22) \times \left((-8) \div \left(\frac{5 + (-7)}{1} \right)^2 \right) \\ & = (-22) \times \left((-8) \div \frac{(-2)^2}{1} \right) \\ & = (-22) \times \left(\frac{(-8) \div 4}{1} \right) \\ & = \frac{(-22) \times (-2)}{1} \\ & = 44 \end{aligned}$$