Lesson 1 - Perfect Squares

Exponents

I want you to multiply 3 by itself 6 times.

$3 \times 3 \times 3 \times 3 \times 3 \times 3$

Is there a better or compact way to write this?

YES ⟶ EXPONENTS will help us do that.

$3 \times 3 \times 3 \times 3 \times 3 \times 3 \longrightarrow 3^6$

this is read as

- " 3 to the 6 "
- " 3 to the exponent 6 "
- " 3 to the power of 6 "

3^6 ⟵ EXPONENT (power)

BASE

The EXPONENT tells us how many times to multiply the BASE to itself.

$4^3 = 4 \times 4 \times 4$

$= 16 \times 4$

$4^3 = 64$

Exponents are a way to save time when you need to multiply the same number multiple of times.

$7^2 = 7 \times 7 = 49$ Did we save time? **NO!**

Use your calculator to answer this multiplication.

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

3 × 3 × 3 × 3 × 3 × 3 = 729

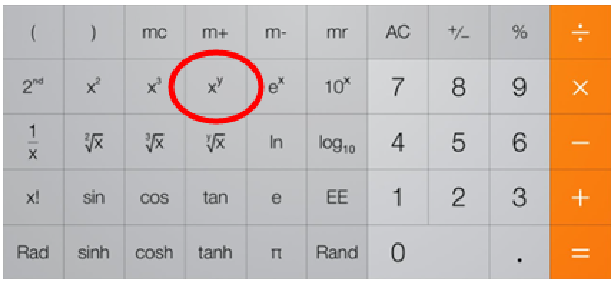
The power of using exponents.

$3^6 \longrightarrow$ 3 x^y 6 = 729

a) $5^7 =$ Answer

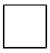
b) $2^{15} =$ Answer

c) $11^5 =$ Answer

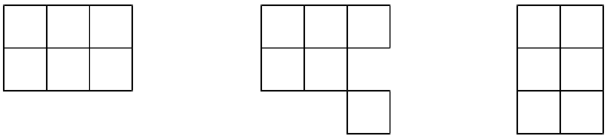


Square Numbers

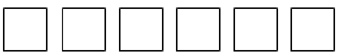
This tile is a square.



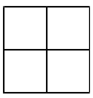
Can we arrange 6 of these identical tiles to make a SQUARE without any gaps?



Of the 6 tiles how many are needed to make a square other than 1 tile?



A solution would be 4 tiles.

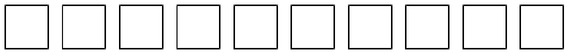


Square Numbers

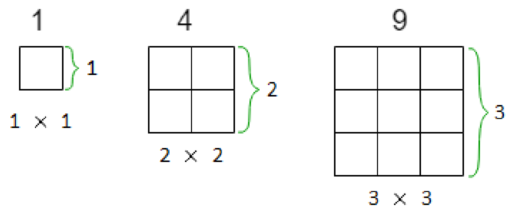
Here we have 10 tiles.

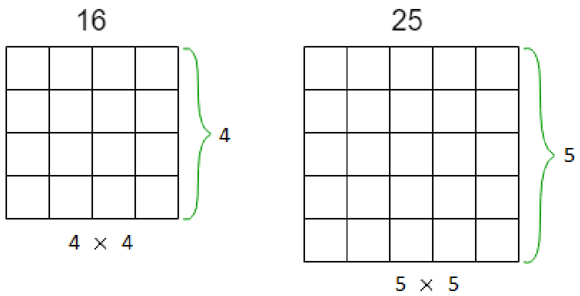
Determine how many squares you can make without any gaps using a maximum of 10 tiles.

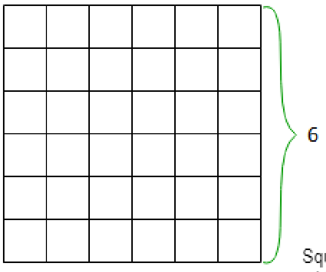
You can use 1 tile or 2, 3, 4 and so on to try and make a square, up to 10 tiles.



If we had access to more tiles what would be the next number of tiles that could be made into a square?







36

$6 \times 6 = 6^2$

six squared

Squaring a number means making a square with a side length of that number.

Definition

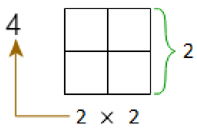
A **square number** is the result of an integer multiplied by itself.
It represents the area of a square with an integer side length.
A square number can also be called a **perfect square**

Square Numbers

A **square number** is the result of an integer multiplied by itself.
It represents the area of a square with an integer side length.
A square number can also be called a **perfect square**

$(\text{positive integer}) \times (\text{positive integer}) = (\text{positive integer})^2 = \textbf{perfect square}$

$2 \times 2 = 2^2 = 4$

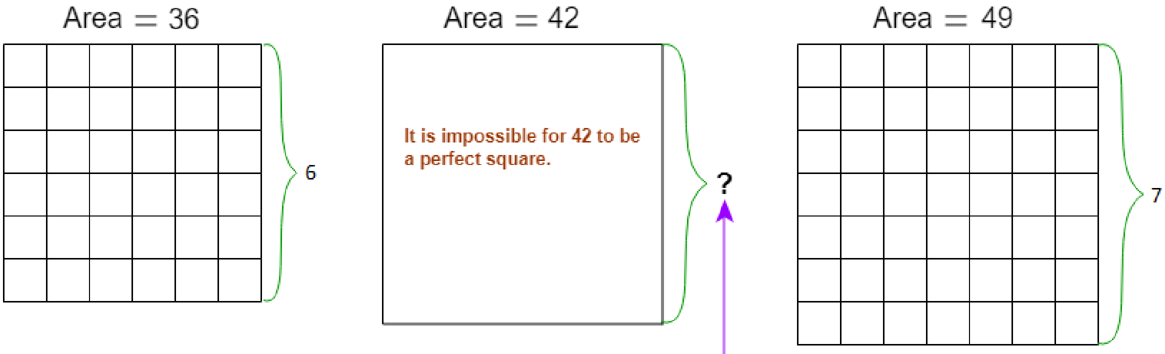


Number	Number Squared	Perfect Square
1	$1 \times 1 = 1^2$	1
2	$2 \times 2 = 2^2$	4
3	$3 \times 3 = 3^2$	9
4	$4 \times 4 = 4^2$	16
5	$5 \times 5 = 5^2$	25
6	$6 \times 6 = 6^2$	36
7	$7 \times 7 = 7^2$	49
8	$8 \times 8 = 8^2$	64
9	$9 \times 9 = 9^2$	81
10	$10 \times 10 = 10^2$	100
11	$11 \times 11 = 11^2$	121
12	$12 \times 12 = 12^2$	144

To do well in this unit and in future math courses then remember these perfect squares.

Perfect Squares

Explain why 42 is not a perfect square?



A **square number** is the result of an integer multiplied by itself.

It represents the area of a square with an integer side length.

A square number can also be called a **perfect square**

A square with an area of 42 has an integer side length?

NO!

Positive Integers

1, 2, 3, 4, 5, 6, 7, 8, . . .

no integer between 6 and 7

There are numbers out there that are not perfect squares.

Perfect Squares

Is 441 a perfect square? Explain why or why not?

$21 \times 21 = 441$ **YES!**

A **square number** is the result of an integer multiplied by itself.

We can easily generate a perfect square.

Take any positive integer , multiply it by itself
and the result is a perfect square.

Let's go the other way.

574 Is 574 a perfect square?

Factors of Perfect Squares can help us.

$36 \longrightarrow 1 \times 36$

2×18

3×12

6×6

$15 \longrightarrow 1 \times 15$

3×5

Factors of 36:

$\underbrace{1,\ 2,\ 3,\ 6,\ 12,\ 18,\ 36}_{\text{7 factors}}$

Factors of 15:

$\underbrace{1,\ 3,\ 5,\ 15}_{\text{4 factors}}$

Perfect squares always have an ODD number of factors.

Non-Perfect Squares always have an even number of factors.

Is 574 a perfect square?

List the factors of 574.

$574 \longrightarrow 1 \times 574$

2×287

7×82

14×41

574 is not a perfect square as there are an even number of factors.

Lesson 2 - Square Roots

Introducing the Square Root

A **square number** is the result of an integer multiplied by itself.

$3 \times 3 = 3^2 = 9$

3 squared

It represents the area of a square with an integer side length.

3 | A = 9

3

The area of a 3 by 3 square is equal to 3^2.

This is true for any number to the power of two.

The area of a 10 by 10 square is equal to 10^2.

Let's go the other way.

What if we are given the AREA of a square and we want to know the side length.

A square has an area of 25 m^2. What is the side length of the square?

A = 25 length = 5 $5^2 = 5 \times 5 = 25$

length = 5

Area = length $\times$ length = length^2 = 25

We must find a positive number that squares to 25.

Do you know what number that is? 5

Mathematicians call this number the **"square root"** of 25

and write this as $\sqrt{25}$ $\sqrt{\ }$ ← radical symbol

Determining the **side length** from a diagram of a **perfect square** is also known as **"Taking the Square Root"** of a number.

Perfect Squares and Square Roots

We know that: $5^2 = 5 \times 5 = 25$

We say that 25 is a perfect square because it is the square of the integer 5.

A square of any integer is a **perfect square**

The first few perfect squares are: 1, 4, 9, 16, 25, 36, . . .

What is the side length of the perfect square 16 ?

"Take the Square Root" of 16

$\sqrt{16} = 4$

4×4

Area = 16 } 4

The length of one side of a square is the "square root" of it's area.

We call 4 the square root of 16 and write $\sqrt{16} = 4$

The radical symbol $\sqrt{\ }$, is used to show a SQUARE ROOT.

To Summarize:

since → $4^2 = 16$

then $\sqrt{16} = 4$

since → $7^2 = 49$

then $\sqrt{49} = 7$

since → $11^2 = 121$

then $\sqrt{121} = 11$

Example Evaluate the following.

a) $\sqrt{144} =$

Answer Solution

b) $\sqrt{81} =$

Answer Solution

c) $\sqrt{1} =$

Answer Solution

Prime Factors to find Square Roots

You should be able to remember the first 12 perfect squares.

$12^2 = 144$

Because you know 144 is a perfect square then you can find the square root of 144.

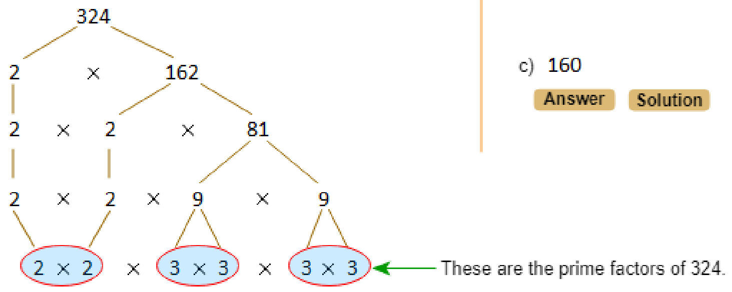
$\sqrt{144} = 12$

324 is a perfect square What is the square root of 324?

We can use prime factorization to find the $\sqrt{324}$.

A **prime number** has only two factors, itself and 1.

Here are the first few prime numbers: $2,\ 3,\ 5,\ 7,$ and 11



$\sqrt{324} = 2 \times 3 \times 3$

$\sqrt{324} = 18$ This method only works if it is a perfect square.

Example

Use prime factorization to determine the square root of the following.

a) 441

Answer Solution

b) 256

Answer Solution

c) 160

Answer Solution

Squaring Negative Numbers

We know that $3^2 = 9$

Can you think of any other number that squares to 9 ?

What do we know about integers.

$3^2 = 9$

$3^2 = 3 \times 3 = 9$

We know two negative numbers multiplied equal a positive number.

$(-3)^2 = 9$

$(-3)^2 = (-3) \times (-3) = 9$

We can conclude that 3 and -3 square to equal 9.

We write $\sqrt{9} = 3$ We can also write $-\sqrt{9} = -3$

Find two integers that square to equal 121.

First integer.

$\sqrt{121} = 11$

$11^2 = 11 \times 11 = 121$

Second integer.

$-\sqrt{121} = -11$

$(-11)^2 = (-11) \times (-11) = 121$

We can conclude that 11 and -11 are the two numbers that square to equal 121.

The positive square root of a number is called the **Principal Square Root**

The symbol $\sqrt{}$ denotes the positive or principal square root.

Therefore $\sqrt{121} = 11$; $\sqrt{81} = 9$

Note: If you want the negative square root you write:

$-\sqrt{121} = -11$

If you want both square roots you write:

$\pm\sqrt{121} = \pm 11$

Square Roots of Fractions

Can a fraction be a perfect square?

$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ 9 is a perfect square. 16 is a perfect square.

$\frac{3^2}{4^2} = \frac{9}{16}$

$\sqrt{\frac{9}{16}}$ ← numerator is a perfect square ← denominator is a perfect square

RULE: Treat the numerator and denominator as SEPARATE perfect squares.

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

$$\sqrt{\frac{9}{16}} = \frac{3}{4}$$

To generate a perfect square we can take the number and multiply it by itself and the answer is a perfect square.

$36 \times 36 = 1296$ ← **perfect square**

Review Square Roots

You have discovered that the square roots of perfect squares are easy.

What is the square root of 49?

$\sqrt{49} = ?$

Ask yourself:
What positive integer multiplied by itself equals 49?

We know that $7 \times 7 = 7^2 = 49$

$\sqrt{49} = 7$ ← Integer

49 is an example of a **perfect square** .

A perfect square is a number that can be expressed as a product of two equal integers.

$16 = 4 \times 4$

$36 = 6 \times 6$

$64 = 8 \times 8$

We might want to find the square root of these numbers.

	2	3		5	6	7	8		10
11	12	13	14	15		17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48		50
51	52	53	54	55	56	57	58	59	60
61	62	63		65	66	67	68	69	70

$\sqrt{29} = ?$

25 26 27 28 29 30 31 32 33 34 35 36

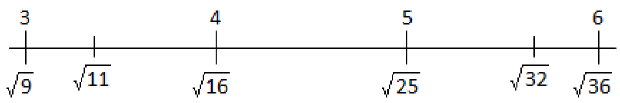
$\sqrt{25}$ ↓ 5 ← $\sqrt{29}$ Is closer to 5 than 6 ↓ ? → $\sqrt{36}$ ↓ 6

The square root of 29 will be a decimal approximation.

Lesson 3 - Estimating and Non-Perfect Root

Estimating Square Roots

Estimating square roots using a number line.



Approximately where would the $\sqrt{11}$ be located?

Approximately where would the $\sqrt{32}$ be located?

Give a reason why where you placed $\sqrt{32}$ on the number line.

The number 32 is 7 units away from 25.
The number 32 is 4 units away from 36.

$\sqrt{11}$ is closer to the $\sqrt{9}$, which is 3 than the $\sqrt{16}$, which is 4

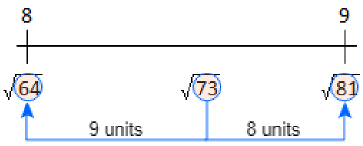
The square root of 11 is closer to 3 than 4.

$\sqrt{32}$ is closer to the $\sqrt{36}$, which is 6 than the $\sqrt{25}$, which is 5

The square root of 32 is closer to 6 than 5.

Which integer is the $\sqrt{73}$ closest to?

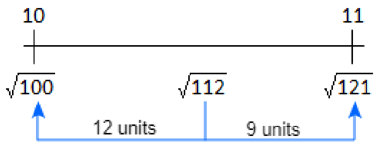
$8^2 = 64$ $9^2 = 81$



The $\sqrt{73}$ is closest to the integer 9.

Which integer is the $\sqrt{112}$ closest to? Give an explanation why.

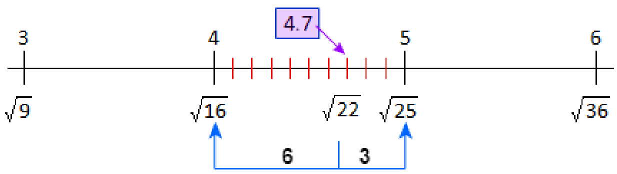
$10^2 = 100$ $11^2 = 121$



The $\sqrt{112}$ is closest to the integer 11.

Estimating Square Roots

Estimating square roots using a number line.



What is the approximate value of $\sqrt{22}$?

Observe: The number 22 is 6 units from 16.
The number 22 is 3 units from 25.

$\sqrt{22}$ is closer to the $\sqrt{25}$, which is 5 than the $\sqrt{16}$, which is 4

The square root of 22 is closer to 5 than 4.

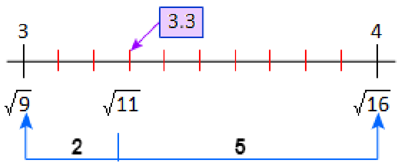
How much closer?

$4.7 \times 4.7 = 22.09$

$\sqrt{22} \approx 4.7$

approximately

What is the approximate value of $\sqrt{11}$?



The square root of 11 is closer to 3 than 4.

$3.3 \times 3.3 = 10.89$

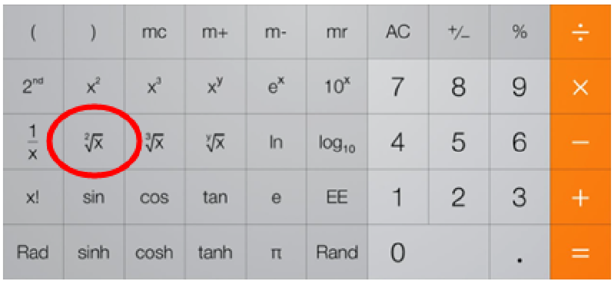
$\sqrt{11} \approx 3.3$

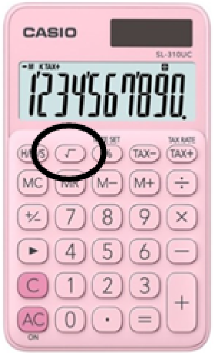
You could have answered: $\sqrt{11} \approx 3.2$ or $\sqrt{11} \approx 3.4$

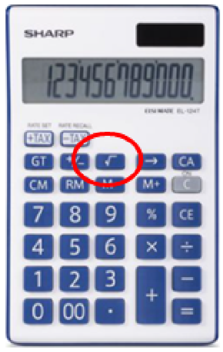
Square Roots using a Calculator

What is the approximate value of $\sqrt{22}$?

Scientific calculators can perform this calculation.







$\sqrt{22} \longrightarrow$ 2 2 $\sqrt[2]{x}$ = **4.69041575982343**

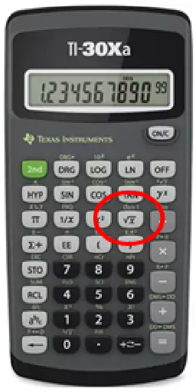
This is more accurate than our estimation in the previous page where

$\sqrt{22} \approx 4.7$

However, 4.69041575982343 is still an approximation of the $\sqrt{22}$

$\sqrt{22} \approx 4.\underbrace{69041575982343}$

This decimal representation will go on forever and never repeat.



To find the square root:

1) Enter the number
2) Press the square root button.

On some calculators:

1) You might press the square root button first,
2) Enter the number
3) Press the equal sign

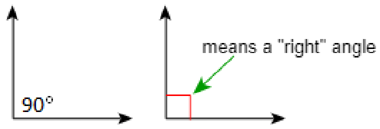
Your calculator will probably give you the answer to 10 or maybe more decimals.

Lesson 4 - Right Triangles and Pythagorean Theorem

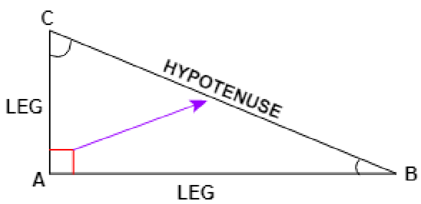
Right Triangles

Recall what a right angle is?

An angle that is equal to 90°.



Right Triangle - one of the angles must be 90°.



In any triangle the longest side will always be opposite the largest angle.

HYPOTENUSE - always opposite the right angle.
- always the longest side.

The other two sides are called the LEGS .

The LEGS always meet at the right angle.

Recall the sum of the three angles in any triangle must be 180°.

If $\angle A$ is 90°, which of the following statements is true about $\angle C$:

a) greater than 90°
b) less than 90° ← √
c) equal to 90°

$$\angle A + \angle B + \angle C = 180^\circ$$

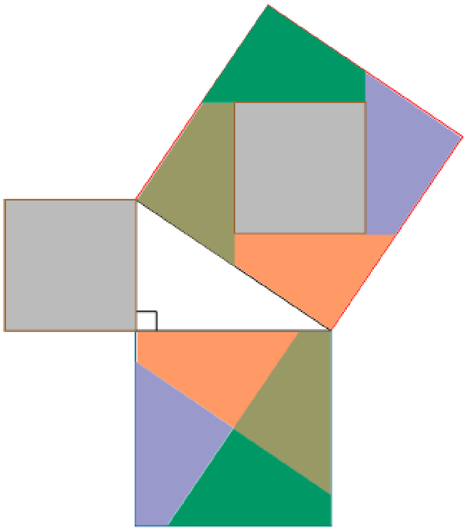
$$\begin{array}{rcl} 90^\circ + \angle B + \angle C & = & 180^\circ \\ -90^\circ \qquad\qquad\quad & & -90^\circ \\ \hline \angle B + \angle C & = & 90^\circ \end{array}$$

We conclude that both $\angle C$ and $\angle B$ must always be less than 90°.

In a right triangle, the 90° angle is always the largest angle.

Squares on the sides of a Right Triangle

Pythagoras was a Greek mathematician and philosopher. He drew squares on the sides of a right triangle to investigate the relationships between the side lengths of the trianlge.



Based on what you have seen, what can you conclude about the squares drawn on the sides of the right triangle?

The five pieces of the area of the two smaller squares fit perfectly inside the largest square.

The Pythagorean Relationship

For a right triangle:

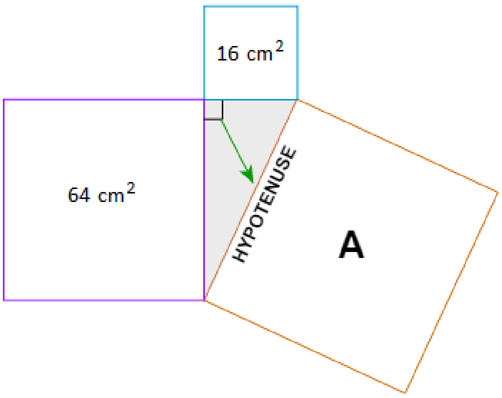
The sum of the area of the squares on the two legs is equal to the sum of the square on the side of the hypotenuse.

Pythagoras discoverd the area of these three squares were related.

The Pythagorean Relationship

The area of the squares of the two legs are $16\ \text{cm}^2$ and $64\ \text{cm}^2$.

What is the area of the square on the hypotenuse?



Let **A** represent the area of the square on the hypotenuse of the right triangle.

By the Pythagorean Relationship:

The sum of the area of the squares on the two legs is equal to the area of the square on the side of the hypotenuse.

$$16\ \text{cm}^2 + 64\ \text{cm}^2 = \mathbf{A}$$

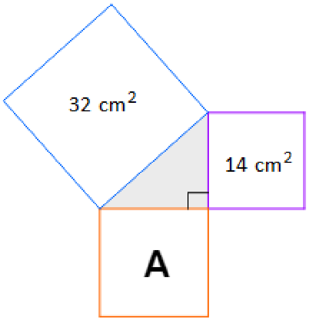
$$80\ \text{cm}^2 = \mathbf{A}$$

Therefore, the area of the square on the hypotenuse is $80\ \text{cm}^2$.

The Pythagorean Relationship

The square on the hypotenuse of a right triangle has an area of $32\ \text{cm}^2$. The square on one leg has an area of $14\ \text{cm}^2$.

What is the area of the square on the leg of the right triangle?



Let A represent the area of the square on the leg of the right triangle.

By the Pythagorean Relationship:

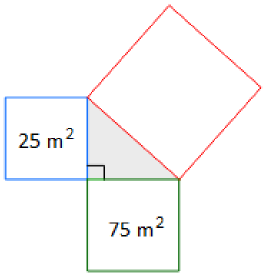
$$\begin{aligned} 14\ \text{cm}^2 + \mathbf{A} &= 32\ \text{cm}^2 \\ -14\ \text{cm}^2 \quad &\quad -14\ \text{cm}^2 \\ \hline \mathbf{A} &= 18\ \text{cm}^2 \end{aligned}$$

Therefore, the area of the square on the leg is $18\ \text{cm}^2$.

The area of the squares of the two legs are $75\ \text{m}^2$ and $25\ \text{m}^2$.

What is the area of the square on the hypotenuse of the right triangle?

(not drawn to scale)



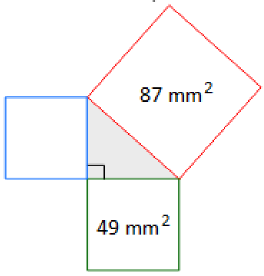
Answer Solution

The square on the hypotenuse of a right triangle has an area of $87\ \text{mm}^2$

The square on one leg has an area of $49\ \text{mm}^2$

What is the area of the square on the remainin g leg of the right triangle?

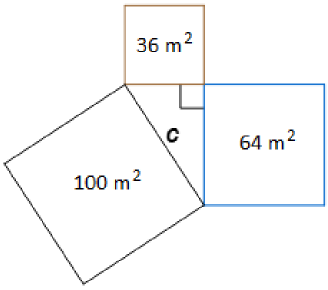
(not drawn to scale)



Answer Solution

The Pythagorean Relationship

The area of the squares on the two legs of a right triangle are $36\ \text{m}^2$ and $64\ \text{m}^2$. What is the length of the hypotenuse?



Let A represent the area of the square on the hypotenuse of the right triangle.

$$36\ \text{m}^2 + 64\ \text{m}^2 = \mathbf{A}$$

$$100\ \text{m}^2 = \mathbf{A}$$ The area is $100\ \text{m}^2$

Find the length of the hypotenuse.

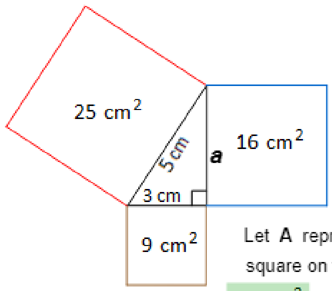
Let $\boldsymbol{c}$ represent the length of the hypotenuse.

$$\boldsymbol{c}^2 = 100\ \text{m}^2$$

$$\boldsymbol{c} = \sqrt{100\ m^2}$$

$$\boldsymbol{c} = 10\ \text{m}$$ The length of the hypotenuse is 10 m

The length of one of the legs of a right triangle is 3 cm. The length of the hypotenuse is 5 cm. What is the length of the remaining leg?



Let A represent the area of the square on the remaining leg.

$$\begin{aligned} 9\ \text{cm}^2 + \mathbf{A} &= 25\ \text{cm}^2 \\ -9\ \text{cm}^2 \quad &\quad -9\ \text{cm}^2 \\ \hline \mathbf{A} &= 16\ \text{cm}^2 \end{aligned}$$

Determine the length of the remaining leg.

Let $\boldsymbol{a}$ represent the length of the remaining leg.

$$\boldsymbol{a}^2 = 16\ \text{cm}^2$$

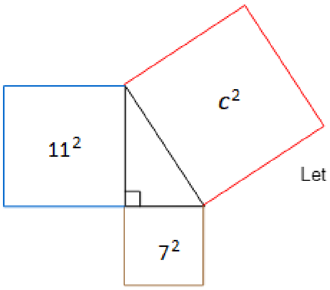
$$\boldsymbol{a} = \sqrt{16\ cm^2}$$

$$\boldsymbol{a} = 4\ \text{cm}$$

The length of the remaining leg is 4 cm.

The Pythagorean Theorem

In a right triangle the lengths of the legs are 11 m and 7 m.
What is the length of the hypotenuse?



Let c represent the hypotenuse.

Using the Pythagorean Relationship:

We know the sum of the area's on the sides of the two legs equals the area of the square on the hypotenuse.

$$11^2 + 7^2 = c^2$$
$$121 + 49 = c^2$$
$$170 = c^2$$
$$\sqrt{170} = c$$
$$c \approx 13.0384\ldots$$

Round answer to two decimal places.

$$c \approx 13.04$$

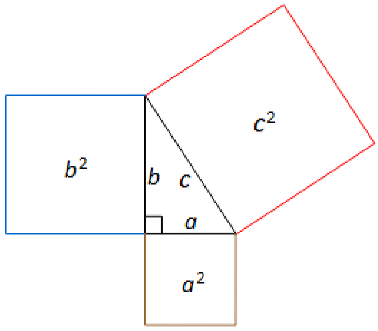
The hypotenuse is aprroximately 13.04 m long.

Re-state the Pythagorean relationship using algebra

The Pythagorean Theorem

In a right triangle where c represents the length of the hypotenuse, and a and b represent the lengths of the two legs, then the following equation is true:

$$a^2 + b^2 = c^2$$

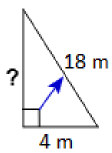


$$\underbrace{a^2 + b^2}_{\text{legs}} = \underbrace{c^2}_{\text{hypotenuse}}$$

The Pythagorean Theorem

Example

In a right triangle, the length of one of the legs is 4 m and the length of the hypotenuse is 18 m. What is the length of the unknown leg?



$$\underbrace{a^2 + b^2}_{\text{legs}} = \underbrace{c^2}_{\text{hypotenuse}}$$

$c = 18$
$a = 4$ m
$b = ?$

$$a^2 + b^2 = c^2$$
$$4^2 + b^2 = 18^2$$
$$\begin{array}{r} 16 \\ -16 \end{array} + b^2 = \begin{array}{r} 324 \\ -16 \end{array}$$
$$b^2 = 308$$
$$b = \sqrt{308}$$

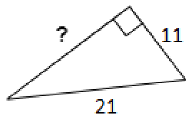
Round answer to 1 decimal place. $b = 17.5499\ldots$

$$b \approx 17.5$$

The length of the unknown leg is approximately 17.5 m.

Example

Find the unkown side in the triangle below. (Round your answer to the nearest tenth).

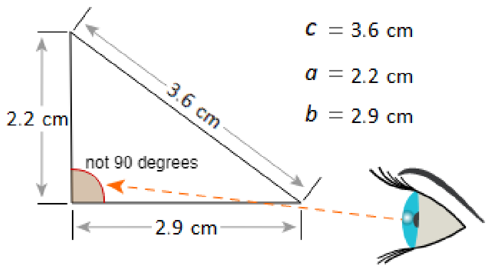


Answer Solution

The Pythagorean Theorem

If	Then
$a^2 + b^2 = c^2$	It IS a Right Triangle
$a^2 + b^2 \neq c^2$	It IS NOT a Right Angle

Is this triangle a right trianlge?



$c = 3.6$ cm

$a = 2.2$ cm

$b = 2.9$ cm

$$a^2 + b^2 = c^2$$

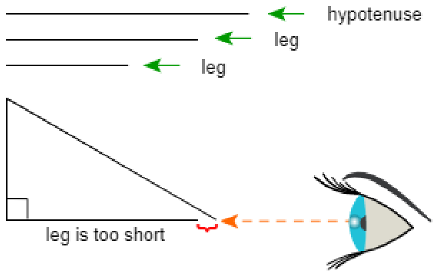
$$2.2^2 + 2.9^2 = 3.6^2$$

$$4.84 + 8.41 = 12.96$$

$$13.25 \neq 12.96$$

This is NOT a right trianlge.

Given these three strings. Can you make a right triangle by connecting the ends of the strings togetther.



Example The sides of a triangle have lengths 5, 11 and 12 cm. Is this triangle a right triangle?

Longest side is the hypotenuse. $c = 12$

Other two lengths are the legs. $a = 5$ $b = 11$

$$a^2 + b^2 = c^2$$

$$5^2 + 11^2 = 12^2$$

$$25 + 121 = 144$$

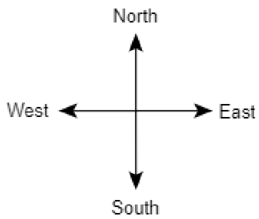
$$146 \neq 144$$

This is NOT a right trianlge.

Lesson 5 - Application of Pythagorean Theorem

Apply the Pythagorean Theorem

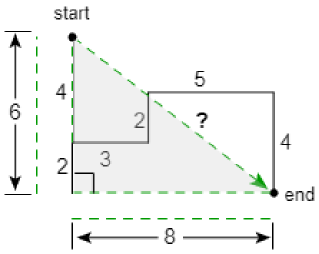
Right triangles and the Pythagorean Theorem play a key role in solving many real world and applied problems.

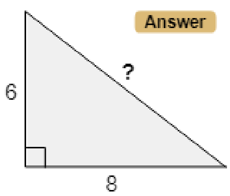


Example

Serge travels 4 km south, 3 km east, 2 km north, 5 km east, then 4 km south. How far did Serge travel in a straight line from his starting point.

Draw a diagram.





Need to know the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

legs hypotenuse

$a = 6$

$b = 8$

$c = ?$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$\sqrt{100} = c$$

$$10 = c$$

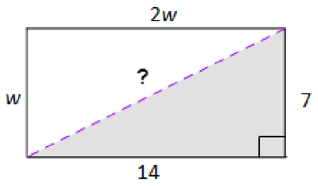
Serge travelled 10 km from the start to the end in a staright line.

Apply the Pythagorean Theorem

Example

A rectangle has a perimeter of 42 cm. If the length is twice as long as it is wide, what is the length of the diagonal of the rectangle?

Draw a diagram.



$$\text{Perimeter} = 42$$
$$2w + w + 2w + w = 42$$
$$\frac{\cancel{6}w}{\cancel{6}} = \frac{42}{6}$$
$$w = 7$$

We now have enough information to find the length of the diagonal.

$a = 7 \quad b = 14 \quad c =$ - diagonal

Note
$a = 14 \quad b = 7$

Can use Pythagorean Theorem

$$a^2 + b^2 = c^2$$

legs   hypotenuse - diagonal

$$7^2 + 14^2 = c^2$$
$$49 + 196 = c^2$$
$$245 = c^2$$
$$\sqrt{245} = c$$
$$c = 15.6524\ldots$$
$$c = 15.7$$

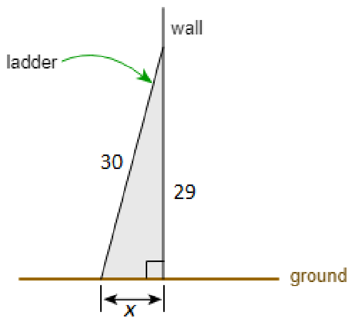
The digonal is 15.7 cm long.

Apply the Pythagorean Theorem

Example

A 30 foot ladder is placed against a wall so that it reaches a height of 29 feet. How far from the base of the wall are the feet of the ladder? Round answer to 1 decimal place.

Draw a diagram.



Can use Pythagorean Theorem

$$a^2 + b^2 = c^2 \qquad a = 29$$
$$b = x$$
$$c = 30$$

$$29^2 + x^2 = 30^2$$
$$\begin{array}{rcl} 841 + x^2 &=& 900 \\ -841 & & -841 \\ \hline x^2 &=& 59 \end{array}$$
$$x = \sqrt{59}$$
$$x = 7.6811\ldots$$
$$x = 7.7$$

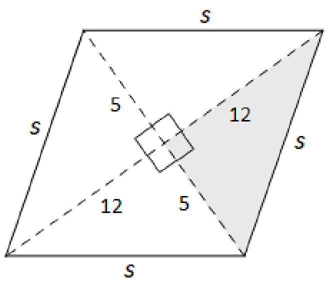
The foot of the ladder is 7.7 feet from the base of the wall.

Apply the Pythagorean Theorem

Example

The length of the diagonals of a rhombus are 10 cm and 24 cm.
What is the perimeter of the rohmbus?

Rhombus



The diagonals of a rhombus bisect each other.

Bisect - cut into two equal parts.

The diagonals intersect at right angles.

Perimeter $= s + s + s + s$

$= 4s \leftarrow$ Need to find the value of s.

Can use Pythagorean Theorem

$$
\begin{aligned}
a^2 + b^2 &= c^2 \qquad a = 5\\
5^2 + 12^2 &= s^2 \qquad b = 12\\
25 + 144 &= s^2 \qquad c = s\\
169 &= s^2\\
\sqrt{169} &= s\\
s &= 13
\end{aligned}
$$

$$
\begin{aligned}
\text{Perimeter} &= 4s\\
&= 4(13)\\
&= 52
\end{aligned}
$$

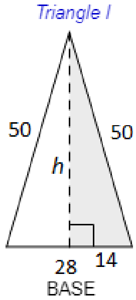
The perimeter of the rhombus is 52 cm.

Apply the Pythagorean Theorem

Example

Of the two triangles below, which triangle has the greater area?

Triangle I



Area of Triangle

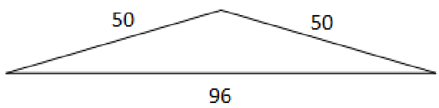
$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$b \rightarrow$ BASE

$h \rightarrow$ HEIGHT

For an isosceles triangle
the height bisects the base.

Trianlge II



Determine the area.

Area of *Trianlge II* $= 672$ **Show Me**

Neither triangles area is bigger than the other.
Both triangles have the same area.

Need to find h

Can use Pythagorean Theorem

$a^2 + b^2 = c^2$

$a = h$

$b = 14$

$c = 50$

$$
\begin{aligned}
h^2 + 14^2 &= 50^2\\
h^2 + 196 - 196 &= 2500 - 196\\
h^2 &= 2304\\
h &= \sqrt{2304}\\
h &= 48
\end{aligned}
$$

Area of *Triangle I*

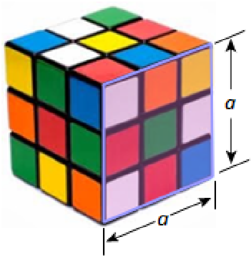
$$
\begin{aligned}
A &= \frac{bh}{2}\\
&= \frac{28(48)}{2}\\
&= \frac{1344}{2}\\
&= 672
\end{aligned}
$$

Lesson 6 - Cube Roots

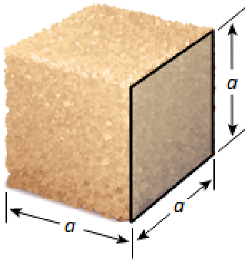
What is a Cube?

Cubic shapes you already know.

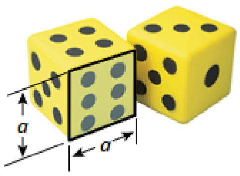
Rubik's Cube



Sugar Cube



Dice



Properties of a Cube

- A cube is a 3-dimensional solid object.
- A side of a cube is called a "face"
- The face of a cube is a SQUARE.
- A cube has six faces.

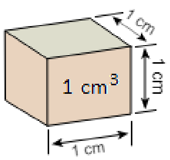
- Volume of a cube.

$V = l \times w \times h$

$= a \times a \times a$

$V = a^3$

Cube Numbers



1 cubic centimeter

Volume of a cube.

$V = a \times a \times a$

$V = a^3$

$V = (1\text{ cm})^3$

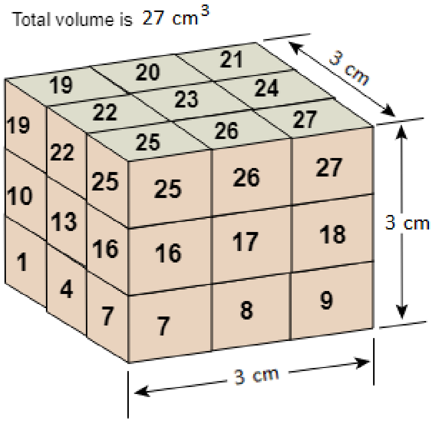
$= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$

$= 1\text{ cm}^3$

This large cube consists of 27 smaller cubes.

What is the volume of the large cube?

Total volume is 27 cm^3



Volume of a cube.

$V = a \times a \times a$

$V = a^3$

$V = (3\text{ cm})^3$

$V = 3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$

$V = 3 \times 3 \times 3\,(\text{cm} \times \text{cm} \times \text{cm})$

$V = 3^3\text{ cm}^3 = 27\text{ cm}^3$

$3^3 = 3 \times 3 \times 3 = 27$

The number 27 is called a PERFECT CUBE.

Perfect Cubes

$3^3 = 3 \times 3 \times 3 = 27$

The number 27 is called a PERFECT CUBE.

$2^3 = 2 \times 2 \times 2 = 8$

$4^3 = 4 \times 4 \times 4 = 64$

$5^3 = \underbrace{5 \times 5}_{25} \times 5 = 125$

25×5

You can easily generate all kinds of perfect cubes.

Simply take any integer and cube it.

$21 \xrightarrow{\text{cube it}} 21^3$

$21^3 = 21 \times 21 \times 21 = 9261$

441×21

The number 9261 is called a PERFECT CUBE.

INTEGER	CUBE IT	EXPANSION	PERFECT CUBE
1	1^3	$1 \times 1 \times 1$	1
2	2^3	$2 \times 2 \times 2$	8
3	3^3	$3 \times 3 \times 3$	27
4	4^3	$4 \times 4 \times 4$	64
5	5^3	$5 \times 5 \times 5$	125
6	6^3	$6 \times 6 \times 6$	216
7	7^3	$7 \times 7 \times 7$	343
8	8^3	$8 \times 8 \times 8$	512
9	9^3	$9 \times 9 \times 9$	729
10	10^3	$10 \times 10 \times 10$	1000

It is a good idea to remember the first 6 perfect cubes.

Cube Roots

Is the number 512 a perfect cube?

What could you do to determine if 512 is a perfect cube or not?

$? \rightarrow ?^3 = ? \times ? \times ? = 512$

We are looking for a number.

Trial and error.

$7 \rightarrow 7^3 = 7 \times 7 \times 7 = 343$

$8 \rightarrow 8^3 = 8 \times 8 \times 8 = 512$

Yes .... 512 is a perfect cube because 8 cubed equals 512

8 is called the "CUBE ROOT" of 512

CUBE ROOTS

Taking the cube root of a number is the opposite operation of cubing a number.

$2 \xrightarrow[\text{cubed}]{2 \times 2 \times 2} 8$

$2 \xleftarrow{\text{cube root}} 8$

Looking for a number such when multiplied by itself three times equals 8.

$2 \xrightarrow[\text{cubed}]{2^3} 8$

$2 \xleftarrow{\text{cube root}} 8$

$\sqrt[3]{8}$

A three in the hook.

Symbol for cube root $\rightarrow \sqrt[3]{}$

Cube Roots

$\sqrt[3]{125} = 5$

Looking for an integer such when multiplied by itself three times equals 125.

$5 \longrightarrow 5^3 = 5 \times 5 \times 5 = 125$

Is -125 a perfect cube?

Is there an integer multiplied by itself three times that equals -125?

$-5 \longrightarrow (-5)^3 = (-5) \times (-5) \times (-5) = -125$

$+25 \times (-5)$

$\sqrt[3]{-125} = -5$

Looking for an integer such when multiplied by itself three times equals -125.

$\sqrt[3]{-64} = -4$

INTEGER	CUBE IT	EXPANSION	PERFECT CUBE
1	1^3	$1 \times 1 \times 1$	1
2	2^3	$2 \times 2 \times 2$	8
3	3^3	$3 \times 3 \times 3$	27
4	4^3	$4 \times 4 \times 4$	64
5	5^3	$5 \times 5 \times 5$	125
6	6^3	$6 \times 6 \times 6$	216
7	7^3	$7 \times 7 \times 7$	343
8	8^3	$8 \times 8 \times 8$	512
9	9^3	$9 \times 9 \times 9$	729
10	10^3	$10 \times 10 \times 10$	1000

Negative Perfect Cubes: -1, -8, -27, -64, -125, -216, -343, -512, -729, -1000

Estimating Cube Roots

You have discovered that the cube roots of perfect cubes are easy.

What is the cube root of 8?

$\sqrt[3]{8} = ?$

Ask yourself:

What integer multiplied by itself three times equals 8?

We know that $2 \times 2 \times 2 = 2^3 = 8$

$\sqrt[3]{8} = 2 \longleftarrow$ Integer

8 is an example of a **perfect cube**.

A perfect cube is a number that can be expressed as a product of three equal integers.

$27 = 3 \times 3 \times 3$

$64 = 4 \times 4 \times 4$

We might want to find the cube root of these numbers.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63		65	66	67	68	69	70

$\sqrt[3]{14} = ?$

8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 27

$\sqrt[3]{8}$ $\sqrt[3]{14}$ $\sqrt[3]{27}$

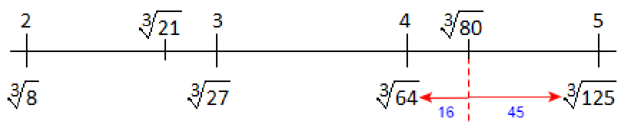
$2 \longleftarrow ? \longrightarrow 3$

Is closer to 2 than 3

The cube root of 14 will be a decimal approximation.

Estimating Cube Roots

Estimating cube roots using a number line.



Approximately where would the $\sqrt[3]{21}$ be located?

Approximately where would the $\sqrt[3]{80}$ be located?

Give a reason why where you placed $\sqrt[3]{80}$ on the number line.

The number 80 is 16 units from 64.
The number 80 is 45 units from 125.

$\sqrt[3]{21}$ is closer to the $\sqrt[3]{27}$, which is 3 than the $\sqrt[3]{8}$, which is 2

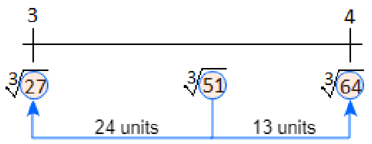
The cube root of 21 is closer to 3 than 2.

$\sqrt[3]{80}$ is closer to the $\sqrt[3]{64}$, which is 4 than the $\sqrt[3]{125}$, which is 5

The cube root of 80 is closer to 4 than 5.

Which integer is the $\sqrt[3]{51}$ closest to?

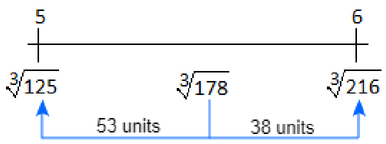
$3^3 = 27$ $4^3 = 64$



The $\sqrt[3]{51}$ is closest to the integer 4.

Which integer is the $\sqrt[3]{178}$ closest to? Give an explanation why.

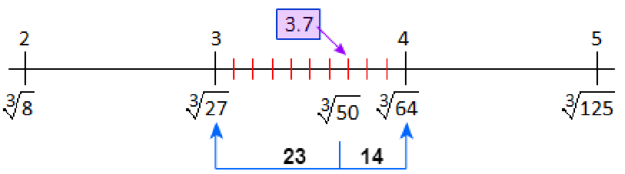
$5^3 = 125$ $11^2 = 121$



The $\sqrt[3]{178}$ is closest to the integer 6.

Estimating Cube Roots

Estimating cube roots using a number line.



What is the approximate value of $\sqrt[3]{50}$?

Observe: The number 50 is 23 units from 27.
The number 50 is 14 units from 64.

$\sqrt[3]{50}$ is closer to the $\sqrt[3]{64}$, which is 4 than the $\sqrt[3]{27}$, which is 3

The cube root of 50 is closer to 4 than 3.

How much closer?

$3.7 \times 3.7 \times 3.7 = 50.653$

$3.69 \times 3.69 \times 3.69 = 50.243409$

$3.68 \times 3.68 \times 3.68 = 49.836032$ ← closer to 50

$\sqrt[3]{50} \approx 3.68$

approximately

What is the approximate value of $\sqrt[3]{470}$?

Answer to two decimal places.

Answer Solution