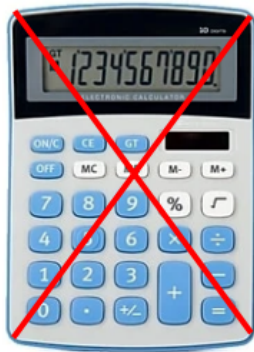


Unit 1 - Lesson 1 - Video Notes

Mental Math



WE CAN GET TO DEPENDENT ON CALCULATORS

Those who manage numbers quickly, (like addition and subtraction) have spent some time practicing **MENTAL MATH**

Example: You are shopping at Canadian Tire and you want to buy a fishing rod that cost \$53 and a tackle box that cost \$39. How much would this cost before taxes?

Think I will add 1 to 39. $53 + 40 = 93$
.....now I must subtract 1 from my answer to get 92.

Quickly add these using the above strategy.

$34 + 18 =$ I will add 2 to 18 **Think** $34 + 20 = 54$

.....now I must subtract 2 from my answer to get the correct answer 52.

- Numbers:**
-we add them $34 + 57$
 -we subtract them $63 - 28$
 -we multiply them 8×7
 -we divide them $42 \div 6$

$$53 + 39$$

$$53 = 50 + 3$$

$$39 = 30 + 9$$

$$80 + 12 = 92$$

$$37 + 26$$

$$7 + 6 = 13$$

$$3 + 2 = 5$$

$$45 + 36$$

$$5 + 6 = 11$$

$$4 + 3 = 7$$

Think $50 + 13 = 63$

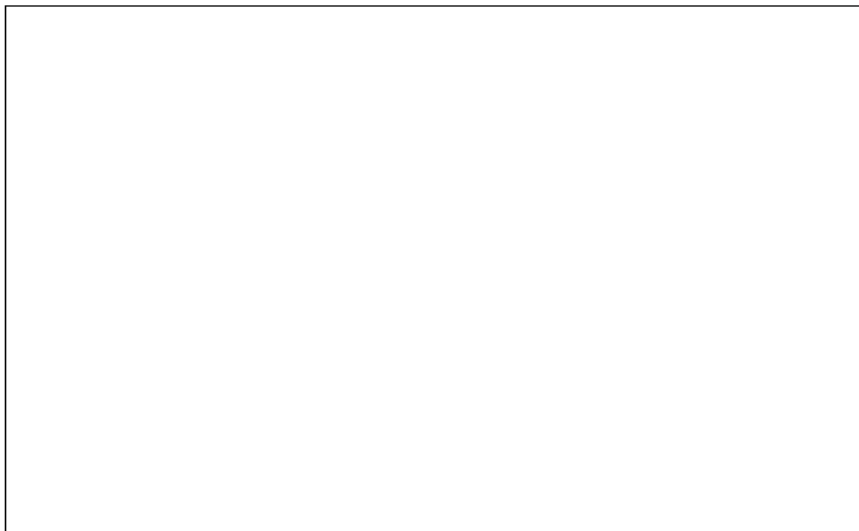
Think $70 + 11 = 81$

Which method works best for you?

Addition Game

Click on the rising balloon that has the result of the given addition.

Score: 0 Level 0 start speed + speed - Speed 1



Mental Math

Adding 3 digit numbers using mental math.

Example: You want to transfer files to your thumb drive that has 835 MB of free space. One file is 453 MB and the other file is 386 MB. Do you have enough space to transfer these files?

$$\begin{array}{r} 453 + 386 \\ \hline \end{array}$$

3 + 6 = 9
5 + 8 = 13
4 + 3 = 7

Think • 700 + 130 + 9
= 839

You do not have enough space on your thumb drive to transfer these files.

$$\begin{array}{r} 294 + 678 \\ \hline \end{array}$$

4 + 8 = 12
9 + 7 = 16
2 + 6 = 8

Think • 800 + 160 + 12
= 972

$$\begin{array}{r} 546 + 729 \\ \hline \end{array}$$

6 + 9 = 15
4 + 2 = 6
5 + 7 = 12 → twelve 100's

Think • 1200 + 60 + 15
= 1275

To ADD 3-digit numbers: 294

100's 1's
 ↑↑↑
 10's

Mental Math

Adding 3 digit numbers using mental math.

$$\begin{array}{r} 480 + 386 \\ \hline \end{array}$$

add 20 ...20 is a friendly number to add

$$\begin{array}{r} 500 + 386 = 886 \\ \hline \end{array}$$

5 + 3 = 8

Think • I added 20 to get this answer.
I must subtract 20 to get the correct answer.
886 - 20 = 866

$$\begin{array}{r} 678 + 294 \\ \hline \end{array}$$

add 6

$$\begin{array}{r} 678 + 300 = 978 \\ \hline \end{array}$$

Think • I added 6 to get this answer.
I must subtract 6 to get the correct answer.
978 - 6 = 972

$$\begin{array}{r} 537 + 768 \\ \hline \end{array}$$

The above strategy is not the best choice to add these numbers.
Not easy numbers to round up to hundreds.

Have to add 32 to get 800.
...then would have to subtract 32 to get the correct answer.

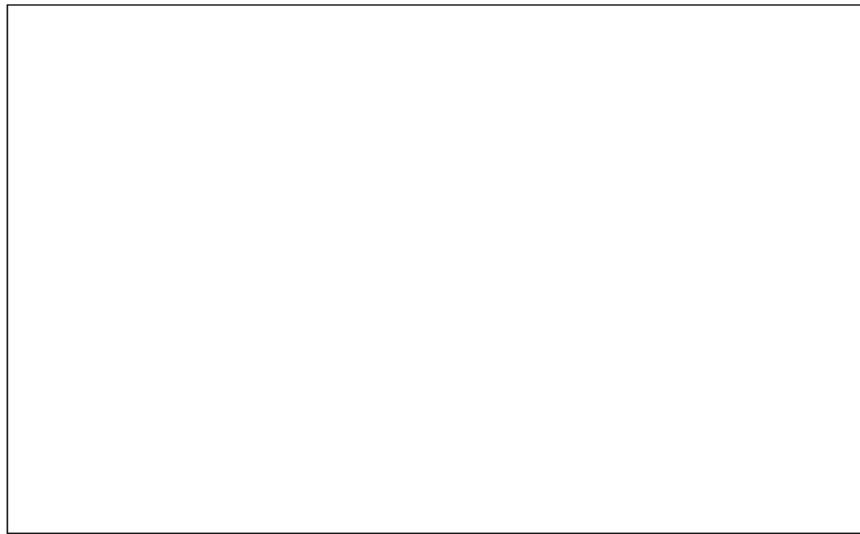
Have to add 63 to get 600.
...then would have to subtract 63 to get the correct answer.

Add nice numbers like:
10, 20, 30
or even
1, 2, 3, 4, 5, 10

Addition Game

Click on the rising balloon that has the result of the given addition.

Score: 0 Level 0 start speed + speed - Speed 1



Subtraction Strategies

Example: You want to purchase a new pair of sweat pants that cost \$53. You check your wallet and have \$85 dollars. How much would you have left if you buy the sweat pants.

$$85 - 53 = 32$$

$$\begin{array}{r} 85 \\ -53 \\ \hline 32 \end{array}$$

Think 8 minus 5 equals 3
5 minus 3 equals 2

$$78 - 24 = 54$$

$$\begin{array}{r} 78 \\ -24 \\ \hline 54 \end{array}$$

Think 7 minus 2 equals 5
8 minus 4 equals 4

These are friendly numbers to subtract.

$$\begin{array}{l} \text{larger} \downarrow \\ 85 - 53 \\ \text{larger} \uparrow \end{array} \qquad \begin{array}{l} \text{larger} \downarrow \\ 78 - 24 \\ \text{larger} \uparrow \end{array}$$

The 10 place value is larger in the first number than the second number

The 1 place value is larger in the first number than the second number

$$63 - 27 = 36$$

$$\begin{array}{r} 5 \cancel{3}^{13} \\ -27 \\ \hline 36 \end{array}$$

$$63 - 27$$

add 3

$$63 - 30 = 33$$

Think I subtracted 3 too many.
I must add 3 to 33 to get the correct answer of 36.

$$74 - 38 = 36$$

add 2

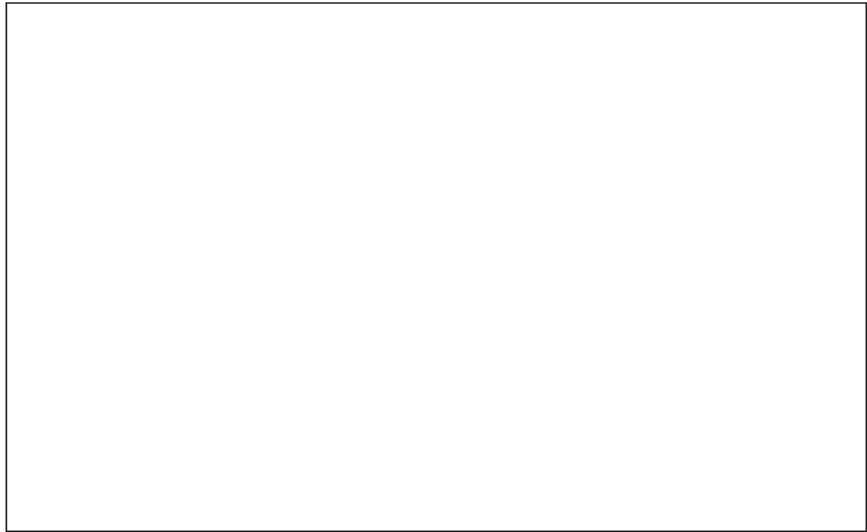
$$74 - 40 = 34$$

Think I subtracted 2 too many.
I must add 2 to 34 to get the correct answer of 36.

Subtraction Game

Click on the rising balloon that has the result of the given subtraction.

Score: 0 Level 0 start speed + speed - Speed 1



Multiplication and Mental Math

Multiply: 34×7



Do you have a strategy to do multiplication in your head?

WE CAN GET TO DEPENDENT ON CALCULATORS


Those who manage numbers quickly, (like multiplying) have spent some time practicing **MENTAL MATH**



Fill in the rest of the times table.
Memorize the Times Table

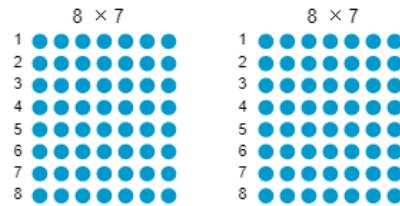
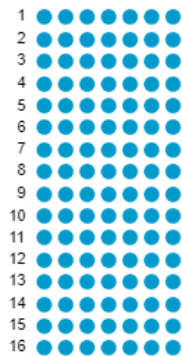
X	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication Strategies

$2 \times 3 = 6$ 

This multiplication is not in your times table.

16×7



two groups of (8×7) means to multiply

$$16 \times 7 = 2 \times (8 \times 7)$$

$$= 2 \times 56$$

$$16 \times 7 = 112$$

This strategy is called HALVING and then DOUBLE

- ▶ Take half of one of the numbers (16)
- ▶ And take that number (8)
- ▶ And multiply it by the second number (7)
- ▶ You must then double the answer (56) to find the answer of the original two numbers.

Use the strategy of halving and double to answer the following multiplications.

a) $18 \times 5 =$

Answer **Solution**

b) $9 \times 12 =$

Answer **Solution**

c) $14 \times 11 =$

Answer **Solution**

NOTE:

This strategy only works well when one of the numbers is even.

Multiplication Strategies

Split Strategy

14×8

$10 + 4$

$10 \times 8 + 4 \times 8$

$80 + 32$

112

Use the Split Strategy to multiply the following in your head.

46×8

$40 + 6$

$40 \times 8 + 6 \times 8$

$320 + 48$

368

Try the following multiplications questions using the Split Strategy

a) $63 \times 6 =$

Answer **Solution**

b) $7 \times 37 =$

Answer **Solution**

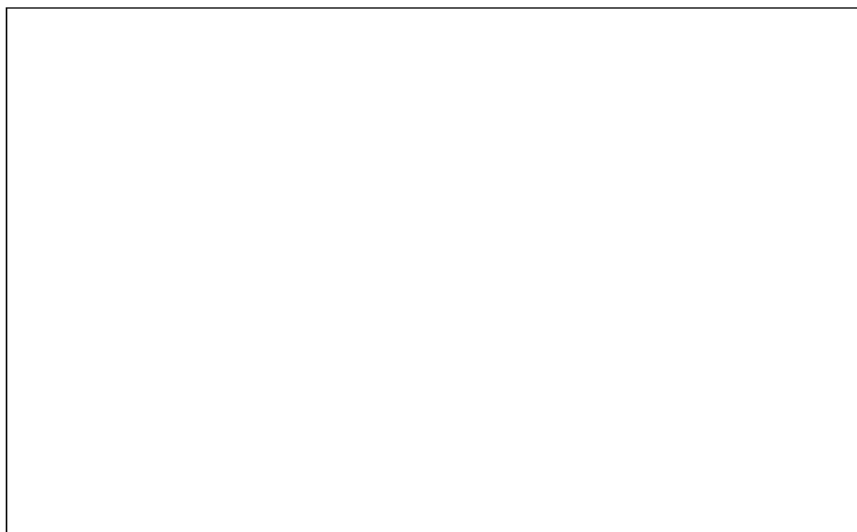
c) $5 \times 86 =$

Answer **Solution**

Multiplication Game

Click on the rising balloon that has the result of the given multiplication.

Score: 0 Level 0 start speed + speed - Speed 1



Multiplication Game

Click on the two numbers that multiply together to give the number shown. You have 90 seconds. Try to improve your score each time.

Time: 00 sec start Score: 0

4	9	9	2	9	4	2	7
5	3	2	6	8	9	4	3
3	8	9	6	3	4	5	5
8	7	5	8	5	8	2	8
2	5	3	4	7	6	6	8
6	8	9	3	5	9	4	4
3	2	7	7	6	5	3	5
9	2	9	5	8	7	9	3

Factors of a Number

Did you know you use factors everyday?




You have a one dollar coin in your pocket and you owe your younger brother 25 cents. You ask your mother to change the dollar into 25 cent coins called a quarter.

How many quarters should your mother give you?

 = 4 × 
 = 20 × 
 = 10 × 
 = 4 × 
 = 2 × 

Factors are useful when travelling.

You plan a trip where the distance is 600 km. How fast and how long do you want to travel?

600 km =  3h × 200 km/h
 600 km =  6h × 100 km/h
 600 km =  30h × 20km/h

Factors of a Number

FACTORS are numbers that you MULTIPLY together to get another number.

$3 \times 6 = 18$
 3 and 6 are **FACTORS** of 18.

$6 \times 5 = 30$
 What are the factors of 30?
 6 and 5 are **FACTORS** of 30.

What are the factors of 12?

Think What two numbers multiplied together equal 12?

- $2 \times 6 = 12$ 2 and 6 are factors of 12
- $3 \times 4 = 12$ 3 and 4 are factors of 12
- $1 \times 12 = 12$ 1 and 12 are factors of 12

The factors of a number always includes 1 and itself.

The factors of 12 are: 1, 2, 3, 4, 6, and 12.

The factors of 12 are: 1, 2, 3, 4, 6, and 12.
all these numbers divide 12

$\frac{12}{2} = 6$ $\frac{12}{4} = 3$ $\frac{12}{12} = 1$

Definition of a FACTOR
A factor is a number that divides into another number exactly and without leaving a remainder.

Determine a factor of 32 other than 1 and 32. Look for a number that divides evenly into 32.

$\frac{32}{4} = 8$

Because 4 divides evenly into 32, then 4 is a factor of 32. Is 8 a factor of 32? Explain why or why not.

Yes $\frac{32}{8} = 4$

$8 \times 4 = 32$

Factors of Large Numbers

Find all the factors of 210.

The first factor that comes to mind is 1.

To organize your work, let's build a table.

When looking for factors they usually come in pairs.

Factor 1	1	2	3	5	6	7	10	14	15	21	30	35	42	70	105	210
Factor 2	210	105	70	42	35	30	21	15	14	10	7	6	5	3	2	1

What do you notice about the list of factors in the top row?

The top row is an INCREASING list of factors.

The bottom row is a DECREASING list of factors.

Therefore, the factors of 210 are:

1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210

The first time you find factors that repeat you can stop.



Think $\frac{210}{2} = 105$

OR
 $2 \times 105 = 210$

Think $\frac{210}{3} = 70$

OR
 $3 \times 70 = 210$

Think $\frac{210}{4}$ 4 does not divide evenly into 210.

$4 \times 5 = 20$
 $4 \times ? = 21$
 $4 \times 6 = 24$

Factors of Large Numbers

a) Find all the factors of 135.

Answer Solution

b) Find all the factors of 324.

Answer Solution

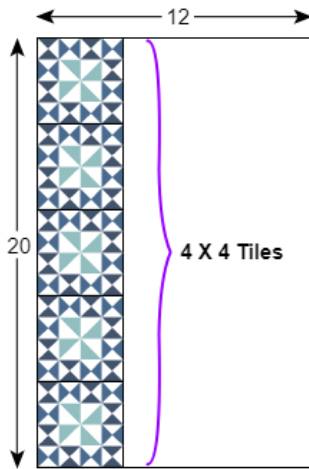
Common Factors

Why do you think it would be useful to know what factors two numbers have in common?

You are building a house and the kitchen is in the shape of a rectangle.

The rectangle is 12 units wide and 20 units long. You want to tile the entire floor with square tiles without making any cuts to the tiles.

What size of square tiles will work?



Let's determine the size of the square tile that will fit along each edge of the rectangle.

Along the side with length 12 units:

Square tile with side length: 1, 2, 3, 4, 6, 12

Notice these are the factors of 12 → 1, 2, 3, 4, 6, 12

Along the side with length 20 units:

Square tile with side length: 1, 2, 4, 5, 10, 20

Notice these are the factors of 20 → 1, 2, 4, 5, 10, 20

Common Factors

We have shown the square tiles that fit along each side of the rectangle.

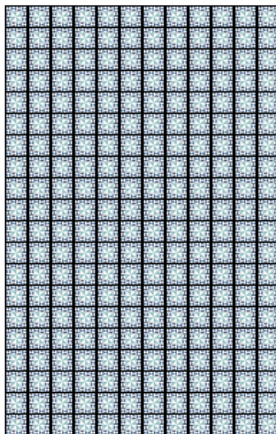
The factors of 12 are: 1, 2, 3, 4, 6, 12

The factors of 20 are: 1, 2, 4, 5, 10, 20

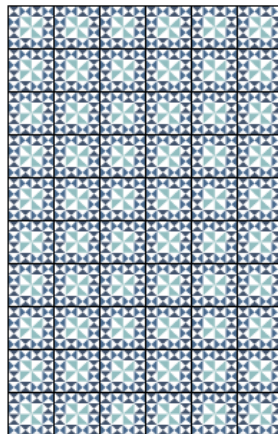
We can tile the floor that is 12 units wide and 20 units long using the following sized square tiles:



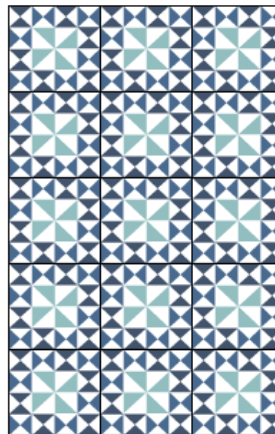
1 X 1 Tiles



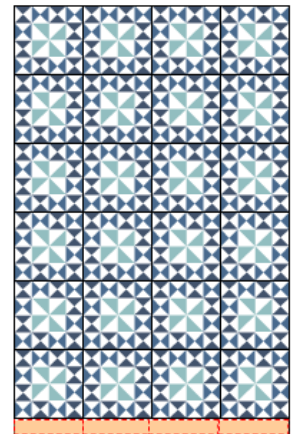
2 X 2 Tiles



4 X 4 Tiles



3 X 3 Tiles



The Common Factors of 12 and 20 are: 1, 2, 4

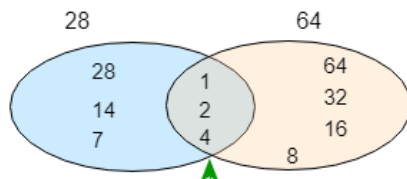
Common Factors

Find all the common factors of 28 and 64.

We need to find the factors of each number and then determine which ones they have in common.

The factors of 28 are:

- 1 and 28
- 2 and 14
- 4 and 7



The factors of 64 are:

- 1 and 64
- 2 and 32
- 4 and 16
- 8 and 8

The common factors of 28 and 64 are: 1 2 4

Example Determine all the common factors of the following pairs of numbers.

a) 18 and 24

Answer **Solution**

b) 64 and 84

Answer **Solution**

Greatest Common Factor (GCF)

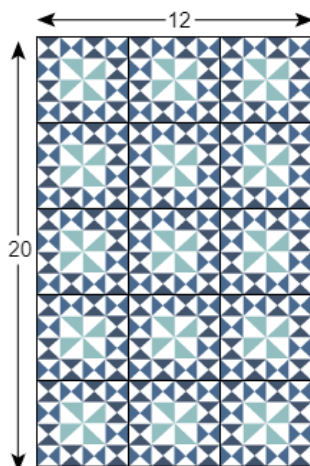
15, 23, 17, 21, 12

All the above numbers have a common factor.

Do you know which factor that is?

1 All numbers are divisible by one.

Often, we are interested in the LARGEST of all the common factors.



1 X 1 2 X 2 4 X 4

The Common Factors of 12 and 20 are: 1, 2, 4.

What tile size is the largest that can be used?

The Greatest Common Factor of 12 and 20 is 4.

4 is the largest number of all the common factors.

Greatest Common Factor (GCF)

Is the largest number that divides evenly into all the numbers in a group of numbers.

(usually a group of two numbers)

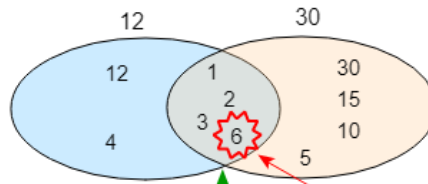
Greatest Common Factor (GCF)

Find the greatest common factor of 12 and 30.

We need to find all the factors of each number.

The factors of 12 are:

- 1 and 12
- 2 and 6
- 3 and 4



The factors of 30 are:

- 1 and 30
- 2 and 15
- 3 and 10
- 5 and 6

The common factors of 12 and 30 are: 1, 2, 3, 6
 The GCF of 12 and 30 is 6.

Example Determine the Greatest Common Factor (GCF) for each of the following pairs of numbers.

a) 24 and 88

Answer **Solution**

b) 18 and 62

Answer **Solution**

Multiples of Numbers

Your school fair has a concession stand and plan to sell hamburgers among other items.

It is your job to buy beef patties and buns to sell at the concession.



1 package = 12 patties

1 package = 8 buns

You want to purchase the SAME number of beef patties and hamburger buns.



1 package = 12 patties

$$1 \times 12 = 12$$



2 packages = 24 Patties

$$2 \times 12 = 24$$



24 is a multiple of 12



3 packages = 36 patties

$$3 \times 12 = 36$$



36 is a multiple of 12

We can only purchase MULTIPLES of 12.

Is 324 a multiple of 12? Explain how you know.

$$\underline{\quad} \times 12 = 324 \quad \frac{324}{12} = 27 \quad 27 \times 12 = 324$$

Yes, 324 is a multiple of 12.

Multiples of Numbers

We can generate multiples of 13.

$1 \times 13 = 13$ Every number is a multiple of itself.

$2 \times 13 = 26$ 26 is a multiple of 13.

$3 \times 13 = 39$ 39 is a multiple of 13.

$10 \times 13 = 130$ 130 is a multiple of 13.

A multiple of 13 is the product of 13 and another number.

List the first 5 multiples of 15.

$1 \times 15 = 15$

$2 \times 15 = 30$

$3 \times 15 = 45$

$4 \times 15 = 60$

$5 \times 15 = 75$

Is it possible to purchase 80 beef patties from the grocery store?



84 beef patties = 7 package of 12 beef patties per pack.

List of multiples: 12, 24, 36, 48, 60, 72, 84

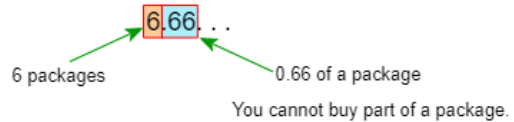
80 is not in the list of multiples

You cannot purchase exactly 80 beef patties.

Is there an easier and quicker way to determine if you can purchase exactly 80 beef patties?

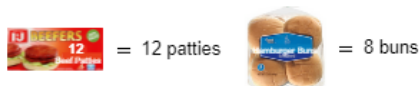
$$\underbrace{80}_{\text{beef patties}} \div \underbrace{12}_{\text{beef patties per package}} = \underbrace{6.66}_{\text{packages}} \dots$$

The answer tells us we would have to buy 6 packages of 12 beef patties and then 0.66 of a package.



Common Multiples

How do we buy equal number of beef patties and hamburger buns?



To solve this problem we need to examine the multiples of 8 and 12 at the same time.

Packages	1 pkg	2 pkgs	3 pkgs	4 pkgs	5 pkgs	6 pkgs	7 pkgs	8 pkgs	9 pkgs
Beef Patties	12	24	36	48	60	72	84	96	108
Buns	8	16	24	32	40	48	56	64	72

In the above example, the numbers 24, 48, and 72 are common multiples of 8 and 12.

By knowing how to generate common multiples you can buy: 6 packages of beef patties (72)

9 packages of buns (72)

Example

a) List the first four common multiples of 4 and 6.

Answer **Solution**

b) List the first four common multiples of 3 and 9.

Answer **Solution**

Least Common Multiple (LCM)

Packages	1 pkg	2 pkgs	3 pkgs	4 pkgs	5 pkgs	6 pkgs	7 pkgs	8 pkgs	9 pkgs
Beef Patties	12	24	36	48	60	72	84	96	108
Buns	8	16	24	32	40	48	56	64	72

Of the above multiples of 8 and 12, what is the smallest common multiple?

The smallest common multiple is 24.

In mathematics the smallest common multiple is known as **Least Common Multiple (LCM)**

We write: $LCM(8, 12) = 24$

Example: Find the Least Common Multiple (LCM) of 4 and 9.

Multiples of 9: 9, 18, 27, 36, 45 . . . $LCM(4, 9) = 36$

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36

A much quicker method.

List the multiples of the largest number first.

Multiples of 9: 9, 18, 27, 36

We can conclude that 36 is a multiple of 4 because: $9 \times 4 = 36$ $LCM(4, 9) = 36$

Think $\frac{9}{4} = 2.25$ $\frac{18}{4} = 4.5$ $\frac{27}{4} = 6.75$ $\frac{36}{4} = 9$ **BINGO!** This is what you want!

Least Common Multiple (LCM)

Example a) Determine the Least Common Multiple (LCM) of 3 and 11.

Answer **Solution**

b) Determine the Least Common Multiple (LCM) of 6 and 13.

Answer **Solution**

Application of LCM

Taylor goes hiking every 8 days and swimming every 6 days. Today he did both . He hiked in the morning and then went swimming in the afternoon. How many days from now will he do both hiking and swimming again?

What We Know

Hikes every 8th day.
Swims every 6th day.

What We Need to Find

In how many days does Taylor do both exercises again.

Strategize and Solve

If the pattern continues (**multiples**) what is the smallest number (**least**) of days until Taylor hikes and swims on the same day. (**common**)

We have to list the multiples of 8 and 6.
Today is day 0.



Hiking	0	8	16	24	32	40	48
Swimming	0	6	12	18	24	30	36

$LCM(6, 8) = 24$

Therefore, Taylor will hike and swim on the same day again in 24 days.

Application of LCM**Example**

At a warehouse distribution facility, the manager prepared for delivery trucks to start arriving after 12 PM. Company ABC delivery trucks were scheduled to arrive every 26 minutes, and XYZ company trucks were schedule to arrive every 18 minutes. How many minutes will pass before two delivery trucks arrive at the same time?

Answer **Solution**

Unit 1 - Lesson 5 - Video Notes

Prime and Composite Numbers

Recall what we mean by a **FACTOR** of a number.

What are the factors of 12?

$$3 \times 4 = 12$$

$$2 \times 6 = 12$$

$$1 \times 12 = 12$$

The factors of 12 are: 1, 2, 3, 4, 6, 12

What are the factors of 13?

$$1 \times 13 = 13$$

The factors of 13 are 1 and 13.

The factors of 12 are: 1, 2, 3, 4, 6, 12

When a number has **EXACTLY** two factors 1 and itself, it is called a **Prime Number**.

When a number has **MORE THAN** two factors it is called a **Composite Number**.

For the following numbers: 2, 3, 4, 5, 6, 7, 8, and 9, classify them as a prime number or a composite number along with their factors in the following chart.

Prime Number	Factors
2	1, 2
3	1, 3
5	1, 5
7	1, 7

Composite Number	Factors
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
9	1, 3, 9

Prime and Composite Numbers

Is 21 a prime number or a composite number?

Prime Number
→ exactly two factors
Composite Number
→ more than two factors

Every number has the factors of 1 and itself.

Some of the factors of 21 include: 1, 21

$$21 \div 3 = 7 \quad 3, 7 \text{ are also factors of } 21.$$

$$3 \times 7 = 21$$

Therefore 21 is a composite number because it has more than two factors.

The factors of 21 are 1, 3, 7, and 21.

Is 31 a prime number or a composite number?

We know 31 has two factors for sure: 1, 31

We must consider if 31 has any factors other than 1 and 31.

~~2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30~~ ...

$$\frac{31}{17} = 1.8235 \dots$$

Therefore, 31 is a prime number because it only has two factors, 1 and itself.

Is 1 a prime number?

When a number has **EXACTLY** two factors 1 and itself, it is called a **Prime Number**.

$$1 \times 1 = 1$$

List the factors of 1: 1

1 has only one factor so it cannot be prime as a prime number has exactly two factors.

Is 1 a composite number? **NO!**

A composite number has more than 2 factors and 1 has only one factor.

Example

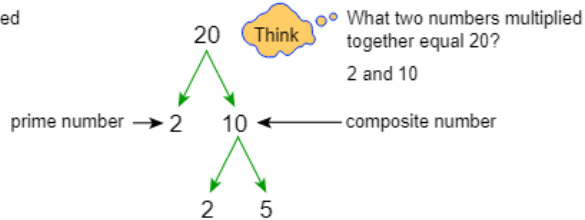
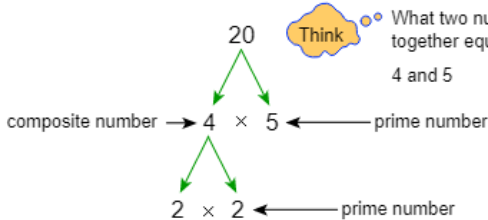
For the list of numbers below, determine if they are prime numbers or composite numbers.

19, 9, 41, 34

Answer

Prime Factorization

Factor Trees



The Prime Factorization of 20

$$20 = 5 \times 2 \times 2$$

The Prime Factorization of 20

$$20 = 2 \times 2 \times 5$$

both have one 5 and two 2's.

A **factor tree** is a diagram that identifies the prime factors of a number.

When we multiply the order of the numbers does not matter.

It does not matter what factors you choose to start with for a factor tree the prime factorization will always be the same.

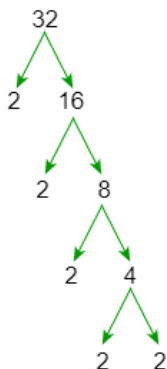
Example

What is the prime factorization of the following numbers.

- a) 56 **Answer** **Solution** b) 84 **Answer** **Solution**

Prime Factorization using Exponents

Using a factor tree, what is the prime factorization of 32?

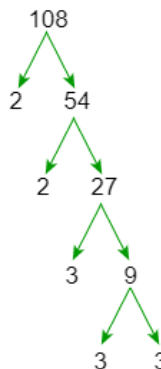


$$32 = \overset{1}{2} \times \overset{2}{2} \times \overset{3}{2} \times \overset{4}{2} \times \overset{5}{2}$$

$$= 2^5$$

Can write this in a compact way using EXPONENTS.

Using a factor tree, what is the prime factorization of 108?



$$108 = \overset{2}{2} \times \overset{2}{2} \times \overset{3}{3} \times \overset{3}{3} \times \overset{3}{3}$$

$$= 2^2 \times 3^3$$

We read this as 2 squared times 3 cubed.

Example

What is the prime factorization of the following numbers. Answer with exponents where possible.

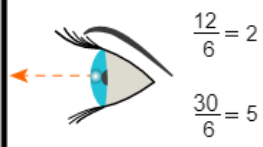
- a) 200 **Answer** **Solution** b) 980 **Answer** **Solution**

Review Greatest Common Factor (GCF)

Greatest Common Factor (GCF)

Is the largest number that divides evenly into all the numbers in a group of numbers.

(usually a group of two numbers)

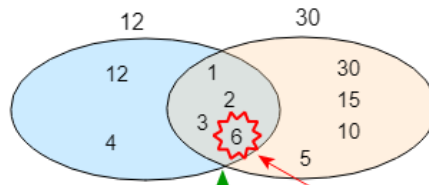


Find the greatest common factor of 12 and 30.

We need to find all the factors of each number.

The factors of 12 are:

- 1 and 12
- 2 and 6
- 3 and 4



The factors of 30 are:

- 1 and 30
- 2 and 15
- 3 and 10
- 5 and 6

The common factors of 12 and 30 are: 1, 2, 3, 6

The GCF of 12 and 30 is 6.

common factors

6 is the largest number of the common factors.

Using Prime Factorization to find the Greatest Common Factor (GCF)

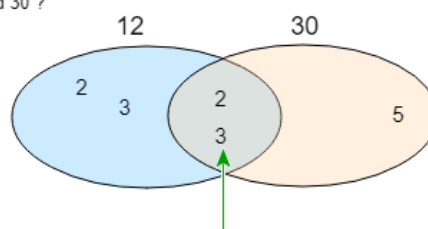
You have learned how to find the prime factorization of a number.

$$12 = 2 \times 2 \times 3 \qquad 30 = 2 \times 3 \times 5$$

What prime numbers are common to both 12 and 30 ?

Place the common prime factors in the overlap.

- One copy of 2.
- One copy of 3.



Multiply the common prime factors to get the Greatest Common Factor (GCF).

$$\begin{aligned} \text{The GCF (12, 30)} &= 2 \times 3 \\ &= 6 \end{aligned}$$

Using Prime Factorization to find the Greatest Common Factor (GCF)

Using prime factorization to find the Greatest Common Factor (GCF) of the following numbers.

a) 30 and 70

Answer **Solution**

b) 16 and 28

Answer **Solution**

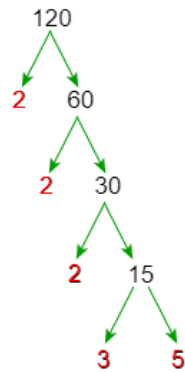
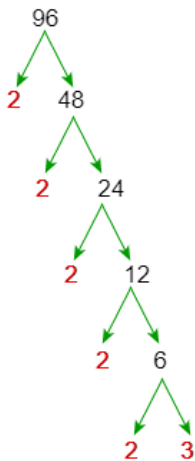
c) 20 and 37

Answer **Solution**

Using Prime Factorization to find the Greatest Common Factor (GCF)

Find the greatest common factor of 96 and 120.

You can use one of two methods to answer this question.



1) List the all factors of each number and determine the largest number that is common to both 96 and 120.

2) Use prime factorization of each number.

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$\begin{aligned} \text{GCF}(96, 120) &= 2 \times 2 \times 2 \times 3 \\ &= 24 \end{aligned}$$

List of factors for 96:

1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

List of factors for 120:

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

Using Prime Factorization to find the Least Common Multiple (LCM)

Least Common Multiple (LCM) \rightarrow Is the SMALLEST positive number that is a multiple of ALL the numbers in the group.

What is the LCM of 4 and 6?

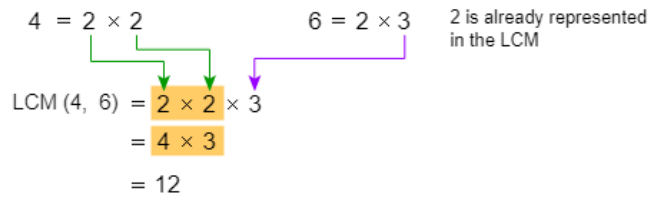
Multiples of 4: 4, 8, 12, 16, 20, 24, ...

Multiples of 6: 6, 12, 18, 24, 30, 36, ...

LCM (4, 6) = 12

Using prime factorization to find the LCM of 4 and 6.

The LCM must contain all the prime numbers from both numbers but no overlap of prime numbers.



Using Prime Factorization to find the Least Common Multiple (LCM)

Listing the multiples of a pair can be time consuming to determine the least common multiple.

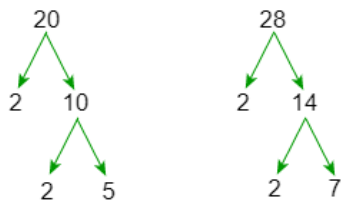
Find the least common multiple of 20 and 28.

Multiples of 20: 20, 40, 60, 80, 100, 120, 140, 160, ...

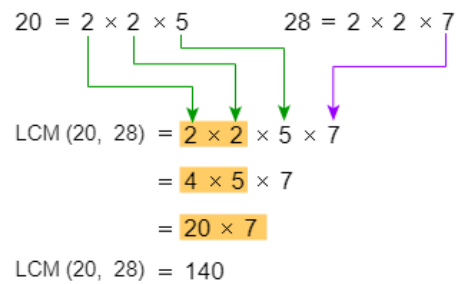
Multiples of 28: 28, 56, 84, 112, 140, 168, 196, ...

Use prime factorization to find the LCM.

Determine the prime factors of 20 and 28 using a factor tree.



The LCM must contain all the prime numbers from both numbers but no overlap of prime numbers.



Using Prime Factorization to find the Least Common Multiple (LCM)

Use prime factorization of the following pairs of numbers to find the least common multiple.

a) 26, 16

Answer **Solution**

b) 12, 28

Answer **Solution**

c) 54, 63

Answer **Solution**

Application of GCF and LCM

Pencils come in packages of 20. Erasers come in packages of 24. Aleah wants to purchase the smallest number of pencils and erasers so that she will have exactly 1 eraser per pencil. How many packages of pencils and erasers should Aleah buy?

What We Know

20 pencils per package
24 erasers per package.

What We Need to Find

The smallest number of packages needed to have 1 eraser per pencil.

Planning to Solve

If there are 20 pencils per package (**multiple**) and 24 erasers per package (**multiple**) and we want the same (**common**) number of pencils and erasers, what is the smallest (**least**) number of packages of pencils and erasers to buy.

We can solve this problem by finding the least common multiple of 20 and 24.

We can use prime factorization to help us find the least common multiple.



$$20 = 2 \times 2 \times 5$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$\begin{aligned} \text{LCM}(20, 24) &= 2 \times 2 \times 5 \times 2 \times 3 \\ &= 120 \end{aligned}$$

Aleah will have to buy:

$$120 \div 20 = 6 \text{ packages of pencils and}$$

$$120 \div 24 = 5 \text{ packages of erasers.}$$

Application of GCF and LCM

Harjit is in charge of making emergency-preparedness kits for his school. He has 72 bottles of water and 90 cans of food, which he would like to distribute equally among the kits, with nothing left over. What is the greatest number of kits Harjit can make?

What We Know

72 bottles of water.
90 cans of food.

What We Need to Find

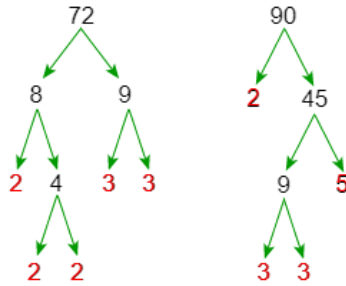
The greatest number of identical kits that can be made.

Planning to Solve

We want to divide (**factor**) the 72 bottles of water and 90 cans of food into identical (**common**) kits to create the largest (**greatest**) number of kits.

We can solve this problem by finding the greatest common factor of 72 and 90.

We can use prime factorization to help us find the greatest common factor.



$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$\begin{aligned} \text{GCF}(72, 90) &= 2 \times 3 \times 3 \\ &= 18 \end{aligned}$$

Therefore, Harjit will be able to make 18 kits composed of:

$$72 \div 18 = 4 \text{ water bottles and}$$

$$90 \div 18 = 5 \text{ cans of food.}$$